







## THE POSITION OF POINTS, LINES AND PLANES IN A MOSQUE CONSTRUCTION FOR GEOMETRICAL VISUALISATION

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**Abstract:** The purpose of this qualitative descriptive study was to identify the concepts of the positions of points, lines, and planes in the building elements of the Kauman *Gedhe* Mosque foyers such as pillars, ceilings, crossbars, and the roof of the building. The research data was obtained from direct observation which was documented using a cellphone camera and visualized with the help of GeoGebra. The results of the study show that there are on the building elements of the Mosque concepts of the position of a point to a line (a point is inside the line and a point is not on the line), a point to a plane (a point is in the plane and a point is outside the plane), a line to a line (two parallel lines, two perpendicular intersecting lines, two non-perpendicular intersecting lines, two coincident lines, and two intersecting lines), a line to a plane (parallel lines, a plane perpendicular, and a plane coincident with a plane), and a plane to a plane (two parallel planes, two non-perpendicular intersecting planes, two perpendicularly intersecting planes, and two coincident planes). Based on the results of this study, it is concluded that there are 16 mathematical concepts found in the *Gedhe* Kauman Mosque. Visualization of abstract mathematical objects can easily be imagined through cultural sites. This is able to lighten cognitive loads so that it is easy to understand, and students are encouraged to construct their own understanding based on the experience they have gained.

**Keywords:** *points, lines, planes, mosques, GeoGebra*

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## INTRODUCTION

The mosque is a religious sacred building that can be found in many countries. As a country with a majority of Moslem population, Indonesia has many historical mosques. The *Gedhe* Mosque of Kauman in Yogyakarta, an Indonesian special province, according to Masfiah (2012), was erected in the reign of the first structural kingdom, commanded by His Highness Sri Sultan Hamengkubowono I on May 29, 1773, architected by Kyai

Penghulu Fakhir Ibrahim Diponegoro and Kyai Wiryokusumo. The mosque is located in the kampong of Kauman, village of Ngupasan, district of Gondokusuman, city of Yogyakarta, and Special Province of Yogyakarta, precisely to the west of the North Yard of the Yogyakarta Palace. The spot on which the mosque is erected is the precise point of the centre of the royal mosque as the royal flagpole of the kingdom vantage point from the five directions of the compass. The mosque is built in the vicinity of the royal palace as a part of the centre of the royal government.

Masfiah (2012) states that the existence of Kauman Gedhe Mosque cannot be separated from the history and struggle of the Indonesian people before independence. The mosque became the living witness of the revision of the direction of the *kiblah* (prayer direction) by K.H. Ahmad Dahlan and MU-APS (Markas Ulama Asykar Perang Sabilillah) [CH-MSSW (Cleric Headquarter of the Moslem Soldiers of the *Sabilillah* War)] during the battle to defend the capital of the state of the Republic of Indonesia when it was moved to Yogyakarta. The mosque, however, has changed functions in accordance with the changes of time. It now primarily functions as a place of worship of the 5-time/day prayers, religious studies, wedding ceremony, bathing dead bodies, religious tourist destination, and the *sekaten* festival.

According to Masfiah (2012), the architecture of the Kauman *Gedhe* Mosque is founded on the concepts of the acculturation of the local cultures and the specific characteristics of the Javanese traditional mosque of the Yogyakarta royal palace patterns. The special characteristics are as follows: the map plan is square in form; the roofs are triple (three-level pyramid); there is a verandah in the front part, one pool in the front yard, and another in the right side; and it is fenced by an inner wall and outer. Then, the mosque has a main prayer chamber housing 900 to 1,000 devotees, supported by 36 pillars made from whole teak wood round in shape and brown in colour. Although the mosque has stood for more than two and a half centuries, these pillars stay intact, no weathering nor eaten by termites. This is because outside the walls, water pools have been built to prevent the wood from being weathered.

The verandah is located lower than the main prayer chamber. The floor is Italian marble. It has two floors; the upper floor has 24 pillars and the lower 32. Each of the pillars stands on a stone pedestal crafted with plant reliefs. On the wall are calligraphies of the Koran verses made of white andesite stone stuck to it without using glue. Decorative ornaments on the pillars and ceiling are dominated with white gold colour with unique royal patterns in green, blue, black, red, and yellow. In addition, chandeliers are hanging on ropes from the ceiling.

From the foregoing description, it can be said, according to the Regional Archives of Yogyakarta ([Dinas Perpustakaan dan Arsip Daerah DIY, 2018](#)), that Kauman *Gedhe* Mosque has an architectural style that is originated from that of Demak Mosque which is characterized by four main pillars called *saka guru* [founding pillar] and 48 other pillars. The main chamber is higher than the verandah, and the verandah is higher than the yard. In the auditorium is found a podium in three pulpits made from teak wood with beautiful ornaments. [Media Museum Nusantara \(2022\)](#) shows that in the north and south parts of the mosque, there is a *pagongan* [music instrument place]; the *Pagongan Ler* [north] is to house the *sekati gamelan* [Javanese music instrument for the *Sekaten* ceremony] Kanjeng Kyai Naga Wilaga; meanwhile, the *Pangongan Kidul* [south] is for the *sekati gamelan* Kanjeng Kyai Guntur Madu.

Mathematics objects, according to [Marsigit, Setiana, & Hardiarti \(2018\)](#), can materially be in the forms of concrete things found in and around the environments, while, formally, they are the processes of the abstraction and idealization of these things. [Zaini & Marsigit \(2014\)](#) state that the use of the concrete models up to the abstract models can give learners the opportunities to develop their thinking, mathematical communication, and creativities. By directly observing mathematics objects, learners will be able to imagine and make relations between elements of a cultural site and abstract mathematical concepts they are learning. This will give experiences to learners so that learning will be more meaningful and enjoyable to them. In the same line, [Muhammad, Marsigit, & Soeharto \(2021\)](#) state that the application of abstract mathematical concepts give advantages in daily lives in various aspects.

Not everybody is aware that the architecture of Kauman *Gedhe* Mosque has geomatric elements in mathematics, specifically in the matters of the positions of points, lines, and planes. [Rosa & Orey \(2013\)](#) state that ethnomathematics is an integration of the cultures of the society, mathematics, and modeling that emerge in the processes of the lives of the society so that they are able to make use of mathematical concepts such as number patterns, geometry, sosial arithmetics, statistics, and so on. It is in this scheme that the Kauman *Gedhe* Mosque can be used as a concrete example of the application of the topics of the standings of points, lines, and planes such that learners, especially of the junior high school (JHS), to understand these concepts better.

The material of the standings of points, lines, and planes is given to the semester-2 JHS students of Year VII. It is a fact that many students have difficultueus in understanding the material. According to [Wulandari & Jailani \(2020\)](#), teachers' obstacles in carrying out the learning processes in the classroom are mostly found in conditioning

the students during the stage in which students ask questions. Students actively ask many and varied questions and teachers need to formulate their answers in the forms of prompting questions in order that students will understand in accordance with the levels of their pre-existing knowledge. In their study, [Sari & Masriyah \(2017\)](#), find a high level of learners' misconception on the instructional material concerning the standings of points, lines, and planes in space using the *Certainty of Response Index* (CRI) method, a gap of 14.81%. In addition, the differences in learners' spatial abilities also cause difficulties in the learners in imagining or visualizing the real forms of abstract mathematical objects. [Khoiriyah, Sutopo, & Aryuna \(2013\)](#) state that learners' level of thinking at the visualizing level (0 level) can only identify the standing of parallel and intersecting lines on pictures; but they are notable to identify the standing of crossing lines.

Previous research on Kauman *Gedhe* Mosque has been done concerning its ornaments, philosophy, and ethnomathematics. [Rohayati, Karno, & Chomariyah \(2017\)](#) studying the geometrical objects in the architecture of the mosque, find information and concepts, among others, about: (1) the natures, area, and circumference of the rhombus of the ornament on the mosque doors; (2) area of the surface and volume of the cube of the royal *maksura* [special chamber for the king to pray]; (3) elements, circumference, and area of the circle on the iron carving on the gate leading to the yard of the mosque, and (4) and the concept of the area of the rectangle cement block used as the floor of the mosque.

[Setyawati \(2016\)](#), shows how the roof top of the mosque, which consists of three layers of cones, signifies the symbolic levels of the relation between man and God. On the first layer, man's level of his relation to God is simple worldly. On the second, it is symbolized that man has started to leave the worldly affairs. And the top cone symbolizes that man has been oriented towards the hereafter. Then, [Syamsiyah & Muslim \(2018\)](#) state that Kauman *Gedhe* Mosque is one that is built in the Javanese style because of the Javanese ways in the use of the structures of the wood materials, the roof headers, room patterns, and decorative ornaments. The ornaments on the pillars are *Padma* [flower], *Saton* [young white rose], *Praban* [triangle, tip up], *Mirang* [cylindrical in shape], *Sorotan* [ray beam], and *Tlancapan* [triangle, tip down]. The Javanese ornaments on the ceiling have the motifs of *banyu tetes* [water dripping], *tlancapan*, *lung-lungan* [heart-shaped leaf], and *nanasan* [pine apple]. For the doors and windows, the ornaments have the shapes of rhombs and floras. And the ornament of the roof shows the symbol of the Yogyakarta kingdom.

These previous research studies mostly discuss ethnomathematics topics using geometrical concepts viewed from the shapes, namely space and plane forms. On the other hand, there are still many other geometrical concepts that can be drawn from the existence of Kauman *Gedhe* Mosque. This is doubled by the notion that students of the junior high school still need concrete objects in order to understand abstract mathematical concepts. The present study is important to be conducted since it discusses geometrical concepts focusing on the standings of points, lines, and planes on the elements that construct the verandah of the mosque complemented with GeoGebra illustrations.

In their ethnomatematics study exploring learning resources that can be found in Kauman *Gedhe* Mosque, [Maululah & Marsigit \(2019\)](#) are able to facilitate mathematics learning that promotes learners' appreciation of the local cultures in the environment as a contextual medium of instruction through artefacts of the inheritance of high Javanese cultures. This is in line with [D'Ambrosio & Rosa \(2008\)](#) who state that knowledge is built by developing the different processes in various sociocultural groups to enable the elaboration and implementation of mathematical skills of calculating, placing, measuring, drawing, representing, playing with, understanding, describing, and showing the relations among concepts through cultural contexts in order to achieve the pre-determined learning objectives.

Another supporting research study on the concepts of geometry in ethnomatematics is done by [Amalia, Syamsuri, & Ihsanudin \(2021\)](#) about the standings of points, lines, planes, structures, and congruency found in the motif of the Krakatoa batik. In this case, the use of the Krakatoa batik in Cilegon, West Java, is elevated as a mathematics learning resource so that instructional activities can be more meaningful for learners through the cultural phenomena that develop around them. Discussion of the research results focuses more on the concepts of geometric transformations such as reflection, shifting, and rotation. In line with this, [Fitriani \(2022\)](#) states that the *Bimbang Gedang* dance involves the mathematical concepts of measurement (measuring, calculating, and identifying) the basic geometrical concepts of angles, lines, planes, and spaces. This study gives a detailed analysis of the kinds of angles that are formed by the movements of the dancer. For example, in the bowl dance, the concept of parallel lines is used when the dancer is going to flip the bowl on her palm, the concept of the perpendicular line is used at the start of the dance, and the concepts of a flat plane in the form of the triangle in the tip of the folded handkerchief and a round circle on the articles used in the dance.

The foregoing discussion of some of the previous research results shows how ethnomathematics can be used as instructional resources and media. Through instructional processes using ethnomathematics, it is possible to enable the learners to explore information and knowledge from the local cultures around that are very close to their daily lives in the perspectives and angles of views of mathematics. [Marsigit, Setiana, & Hardiarti \(2018\)](#), state that ethnomathematics-based mathematics learning in elementary and junior-high schools can elevate students' abilities in understanding, critical thinking, and constructing concepts and structures in mathematics.

Furthermore, by integrating mathematical ideas and concepts in cultures, students will be able to improve their understanding and creativities in doing mathematics. This is supported by [Suherman & Vidákovich \(2022\)](#) who conduct mathematics learning through the approaches to realities of the regional cultures, in this case: the geometrical concepts found in the Lampung *tapis* [traditional rug, cloth, floor covering]. From the study, it is found that the *tapis* has the geometrical concepts of shifting, rotation, reflection, and dilation. The abilities of the students to identify the flat structures in the *tapis* motif and then to reconstruct it into other variations show that the ethnomathematics approach to the teaching of mathematics is able to develop creativities.

Then, from the study by [Faiziyah et al. \(2020\)](#), it can be shown that there is an increase of 17,91% in the learners' creativity after attending an ethnomathematics instructional program (for the junior vocational school). The evaluation of creativity is done through the students' work sheets focusing on the material of geometric transformation. In the study, the student research subjects carry out an observation task on the motif of the *Slobog* batik and then complete the ensuing instructions.

Geometry becomes part of mathematics where the the existence of the realization of the implementation of elements, contexts, and concepts can easily be found. The learning of the geometrics material using ethnomathematic approaches through cultures becomes one of the methods in mathematics learning which is based on the environment around. As in other methods or approaches, in order to support the ethnomathematics, instructional media are needed. There are a variety of media that can be used for this purpose. The present study makes use of the GeoGebra software as a medium and learning aid in constructing abstract geometric concepts. According to [Asngari \(2015\)](#), use of the features in this software gives ease in explaining geometric objects effectively. Ease in use and its applicative nature make GeoGebra become an alternative to be used to visualize the results of direct observations concerning the concepts of the standings of points, lines, and planes found in the verandah of Kauman *Gedhe* Mosque.



Integration between the awareness and understanding of the geometrical concepts and its existence and application in the environment (including cultures) becomes one of the stimuli in the active learning for the learners. [Sarwoedi, et al. \(2018\)](#) state that, by applying ethnomathematics, teachers can help the learners to improve the learners' abilities in understanding, identifying, and interpreting mathematical concepts and ideas. This is in accordance with the implementation of the Freedom Curriculum ([Pusat Asesmen dan Pembelajaran, 2021](#)) who emphasize on the differentiation learning and character profile of Pancasila learners. Ethnomathematics is relevant with the second dimension of Pancasila learners, namely: "Global Diversity" to guard noble cultures to grow the feelings of knowing and appreciating cultures, the abilities of inter-cultural communication in interacting with others, and the reflections and responsibilities towards diverse experiences.

Use of the local cultures in integration with mathematics learning is believed to challenge learners' curiosities and creativities so that they can have experiences that are meaningful, useful, and saved in their long-term memories. By visualizing the standings of points, lines, and planes, learners will be able to minimize extrinsic cognitive loads. In this case, [Retnowati \(2017\)](#) states that, in the theory of cognitive loading, a successful instruction is one that is able to minimize extraneous cognitive contents and maximize germane cognitive contents, that is, the processing of knowledge construction that is in coherence with the needs and characteristics of the learners. Visualization of abstract objects in the real world helps in speeding cognitive loading such that learners will understand the materials more easily and are better facilitated in constructing their own understanding based on the experiences they obtain.

Based on the foregoing background information, the present study is intended to explore and identify an implementation of the concepts of the standings of points, lines, and planes that can be found in the elements of the verandah of Kauman *Gedhe* Mosque by ways of direct observation and documentations to be then visualized by using the *GeoGebra software* program. Not much ethnomathematical analysis has been done on this topic of the positions of the standings of points, lines, and planes on the research object.

In some previous studies, only a small amount of information is given concerning this topic. Besides, no study is found that uses the visual aid of *GeoGebra* to specifically show artificial objects that represent the mathematical abstract objects on the architecture of the mosque verandah. It is expected that this undertaking will give interest to students in participating in the classroom instruction and simultaneously learning about local

cultures. Information from or results of this ethnomathematical study will be used by mathematics teachers as resources in the teaching of mathematics to semester-2 of the Year-VII students of the junior high school in the topic of the standings of points, lines, and planes.

## METHOD

The study was descriptive qualitative conducted at the beginning of 2023 in the *Gedhe* Mosque of Kauman, Yogyakarta. The mosque stands in the environment of the royal North Park of the royal Palace of Yogyakarta kingdom. The entity is located on Kauman Street, the village of Ngupasan, district of Gondomanan, city of Yogyakarta, province of the Special Territory of Yogyakarta, 55132. Data were collected by direct observation on the interior parts of the verandah of the mosque and recorded and documented by using a cellphone.

## RESULTS AND DISCUSSION

Generally, students have learned about the preliminary knowledge on the concepts of points, lines, and planes, in their primary-school years. Analysis of the data from the observation begins by representing the architectural objects in the mosque verandah such as the lamps, pillars, ceiling, stairs, and walls with mathematical objects such as points, lines, and planes. Representing the objects will make it easy to explain the relations of the standings of the points, lines, and planes. In this case, if the ethnomathematic objects are viewed from a distance, (1) the light bulbs will be seen as points; (2) the pillars as lines; and (3) the ceiling, stairs, walls, and floors as planes.

The GeoGebra is used by teachers as an instructional medium to visualize concrete objects in the illustrative forms as points, lines, and planes. The purpose of re-drawing the ethnomathematic objects using the GeoGebra is to make it easy for the learners in constructing the geometric concepts such that they will be able to visualize the objects into abstract objects. Based on the representing, it can be shown directly the forms that resemble mathematical objects using additional information in order to identify the standings of the objects, as it is shown in [Table 1](#), [Table 2](#), and [Table 3](#).


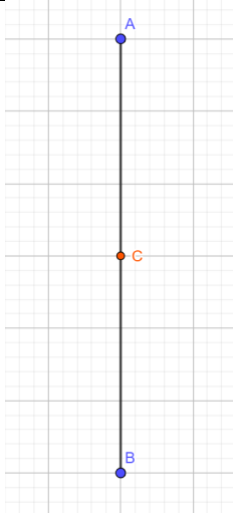
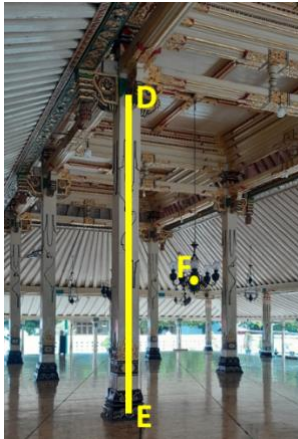
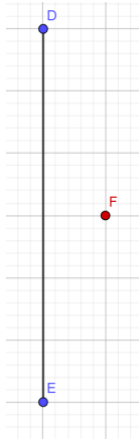
On the roof part of the verandah, there is a girder beam (crossbar) with decorations and carvings of the royal styles. Suppose the beam is the segment of the line AB and the design centre of the blue object is point C. Then, the segment of line AB intersects with point C or point C can be said to lie on the segment of line AB ([Figure 1](#),



Figure 2). Then, in the middle part of the verandah, there are 6 main supporting pillars completed with chandeliers hanging from the ceiling.

Suppose the pillar is the segment of line DE and the light bulb is point F. Then, the line segment of DE does not intersect with point F or it can be said that point F does not lie on the line segment DE (Figure 3, Figure 4). And, on the ceiling of the verandah, there are three main rectangles decorated by carvings of the gold-white nuances, the centre of the ceiling is used to hang the chandeliers, and this centre is point K. Then, point K lies in the rectangle plane GHIJ or it can be said that point K lies on the rectangle plane GHIJ (Figure 5, Figure 6). Then, suppose the ceiling is the rectangle plane LMNO and the chandelier is point P. Then, point P lies outside the rectangle plane LMNO or it can be said that point P does not lie on the rectangle plane LMNO (Figure 7, Figure 8).

**Table 1. Positions of Points**

Position	Document	GeoGebra Illustration
Point onto line (Point lies on line)		
Point onto line (Point not lies on line)		


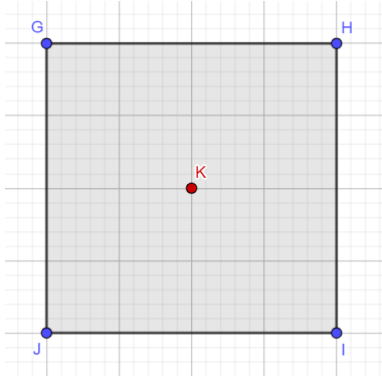
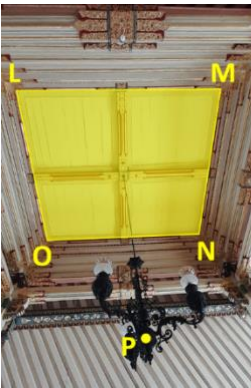
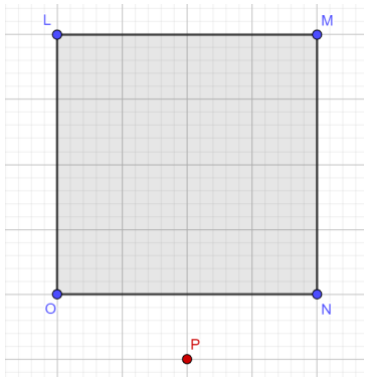
**Figure 1. Point lies on line**

**Figure 2. GeoGebra illustration point not lies on line**

Point onto line  
(Point not lies on line)

**Figure 3. Point not lies on line**

**Figure 4. GeoGebra illustration point not lies on line**

Position	Document	GeoGebra Illustration
Point onto plane (Point lies in plane)		
Point onto plane (Point lies outside plane)		

**Figure 5. Point lies in plane**

**Figure 6. GeoGebra illustration point lies in plane**

**Figure 7. Point lies outside plane**

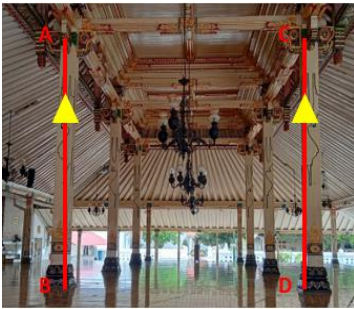
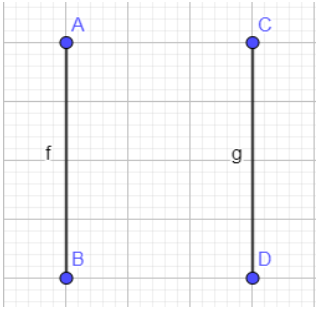

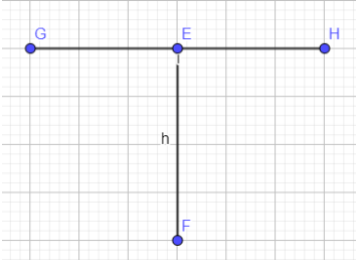

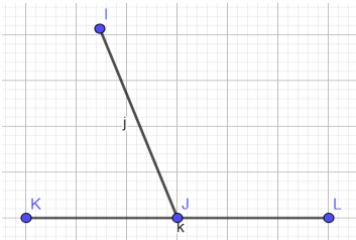
**Figure 8. GeoGebra Illustration Point lies outside plane**

In [Figure 9](#) and [Figure 10](#), segments AB and CD represent mosque pillars. Segments AB and CD are parallel; it can be rewritten as  $AB \parallel CD$ . This is because the two lines AB and CD do not have an intersecting point. In [Figure 11](#) and [Figure 12](#), the line segment EF represents a pillar and GH represents a crossbar. The two segment lines EF and GH intersect at point E. This is because the two intersecting lines make an angle of  $90^\circ$ ; so, they are perpendicular against each other and can be represented as  $EF \perp GH$ .

In [Figure 13](#) and [Figure 14](#), the line segment IJ represents the roof frame of the mosque and the line KL represents the crossbar. The line segments IJ and KL intersect at point J. Because the intersecting IJ and GH make an angle of  $70^\circ$ , they are not perpendicular. Then, in [Figure 15](#) and [Figure 16](#), the line MN represents a pillar and OP represents a crossbar. The line segment MN lies on the floor and ceiling of the mosque while the line segment OP on the ceiling. These two lines lie on different planes and do not have intersecting points. Therefore, lines MN and OP do not intersect.

In [Figure 17](#) and [Figure 18](#), the line segments QR and ST represent crossbars across the ceiling of the mosque. The two lines coincide with each other because they both lie on the ceiling. Then, in [Figure 19](#) and [Figure 20](#), line UV represents the crossbar that connects pillars and the plane ABCD represents the mosque floor. Line UV and plane ABCD can be said to be parallel because they do not make an intersection point and rewritten as  $UV \parallel ABCD$ .

**Table 2. Positions of Lines**

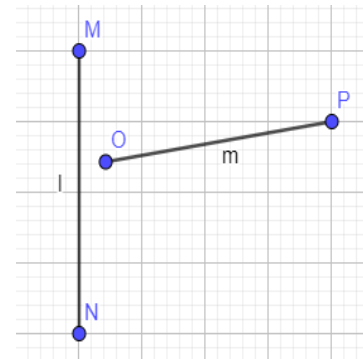
Position	Document	GeoGebra Illustration
Line onto other lines (Two lines perpendicularly intersecting)	 <p><b>Figure 9. Two parallel lines</b></p>	 <p><b>Figure 10. GeoGebra illustration two parallel lines</b></p>
Line onto other lines (Two lines perpendicularly intersecting)	 <p><b>Figure 11. Two lines perpendicularly intersecting</b></p>	 <p><b>Figure 12. GeoGebra illustration two lines perpendicularly intersecting</b></p>
Line onto other lines (Two lines not perpendicularly intersecting)	 <p><b>Figure 13. Two lines not perpendicularly intersecting</b></p>	 <p><b>Figure 14. GeoGebra illustration two lines not perpendicularly intersecting</b></p>

Position	Document	GeoGebra Illustration
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Line onto other lines  
(Two intersecting lines)



**Figure 15.** Two intersecting lines

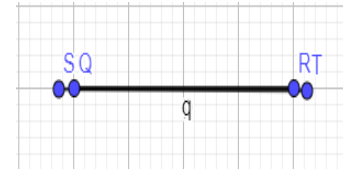


**Figure 16.** GeoGebra Illustration Two intersecting lines

Line onto other lines  
(Two coincident lines)

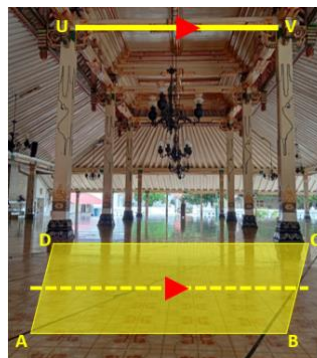


**Figure 17.** Two coincident lines

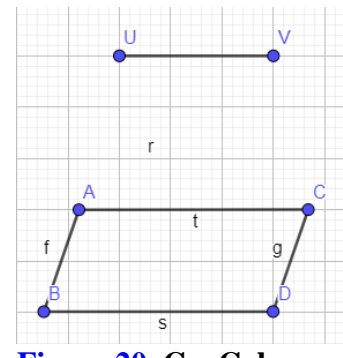


**Figure 18.** GeoGebra Illustration Two coincident lines

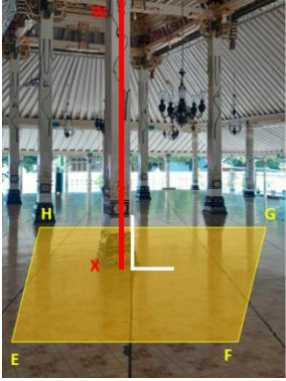
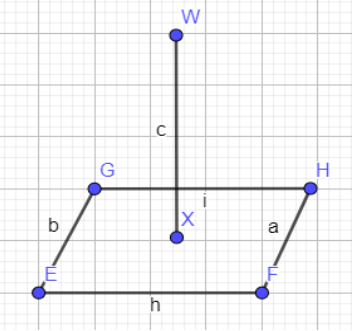
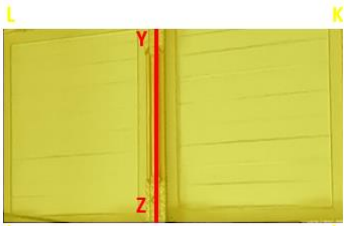
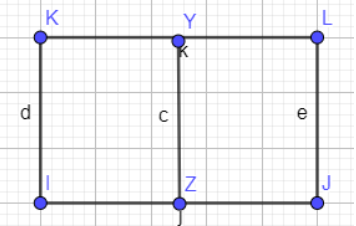
Line onto plane  
(Line parallel with plane)



**Figure 19.** Line parallel with plane



**Figure 20.** GeoGebra illustration line parallel with plane

Position	Document	GeoGebra Illustration
Line onto plane (Line perpendicularly intersecting plane)	 <p data-bbox="608 607 852 707"><b>Figure 21. Line perpendicularly intersecting plane</b></p>	 <p data-bbox="991 544 1286 689"><b>Figure 22. GeoGebra illustration line perpendicularly intersecting plane</b></p>
Line onto plane (Line coincident plane)	 <p data-bbox="608 992 834 1066"><b>Figure 23. Line coincident plane</b></p>	 <p data-bbox="991 992 1286 1097"><b>Figure 24. GeoGebra illustration line coincident plane</b></p>

In [Figure 21](#) and [Figure 22](#), the line segment WX represents a pillar and the plane EFGH the mosque floor. Line WX and plane EFGH intersect at X. Because the intersecting WX and EFGH make an angle of  $90^\circ$ , they are perpendicular and can be represented as  $WX \perp EFGH$ . Then, In [Figure 23](#) and [Figure 24](#), line segment YZ represents the crossbar on the mosque ceiling and the plane IJKL the roof. YZ lies on IJKL so that line YZ is attached to IJKL.

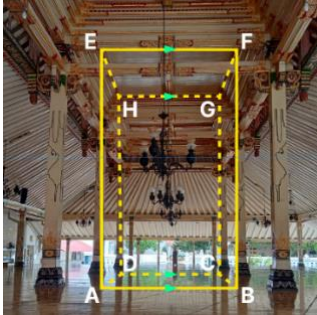
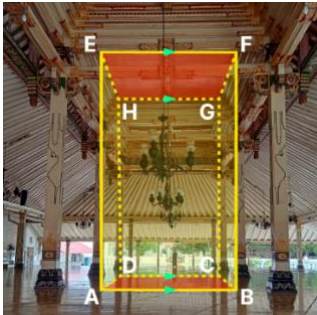
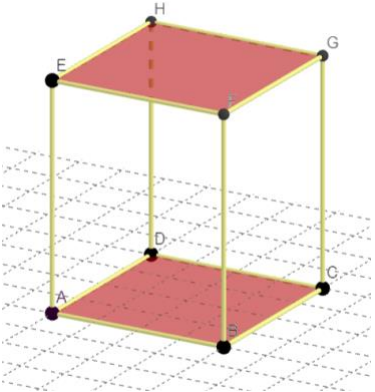

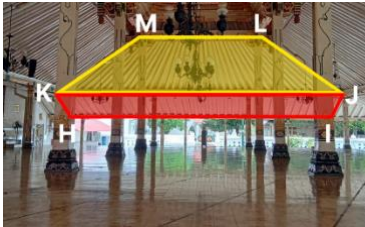
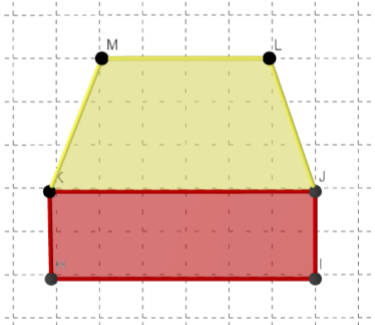
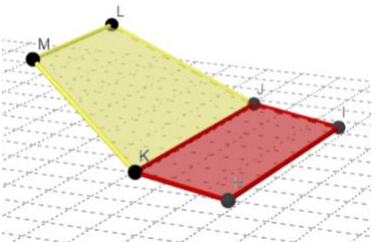
The roof and floor of the mosque verandah, as can be seen in [Figure 25](#), show an application of the concepts of parallel planes. The plane ABCD which represents the mosque floor and EFGH representing the roof are rectangular. The two planes do not have any unifying point, or they do not intersect with each other. So, the planes ABCD and EFGH are parallel, or it can be rewritten as  $ABCD \parallel EFGH$ . If the concept of the parallel planes of the roof and floor of the mosque is illustrated by GeoGebra, the construction can be seen as in [Figure 26](#).

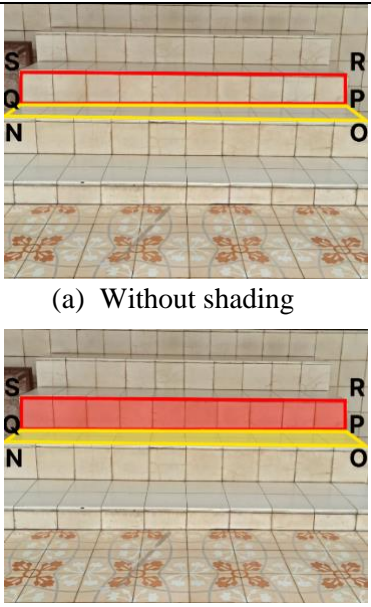
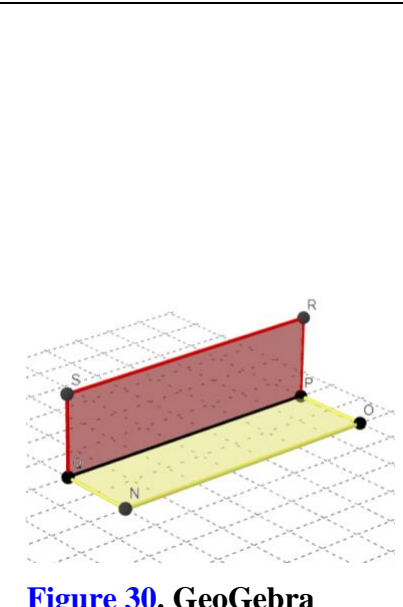
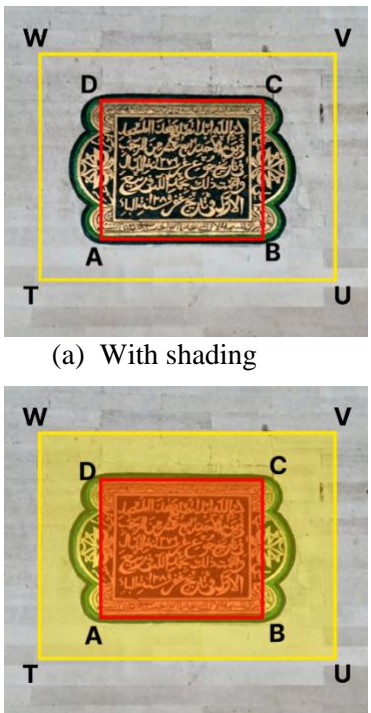
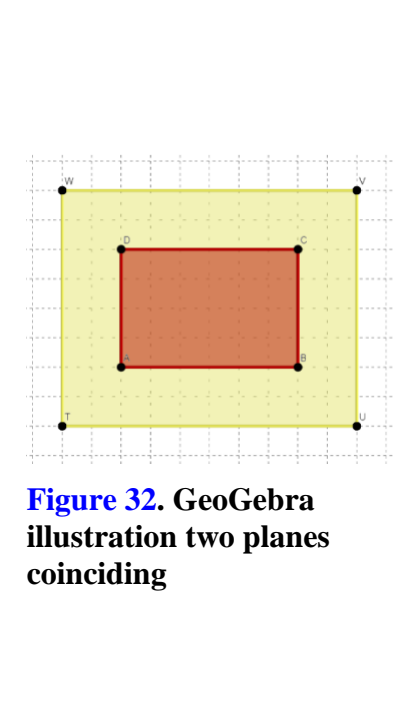
Other than the concept of parallel planes, in the mosque verandah can also be found that of two others, namely unperpendicular intersecting planes and perpendicular intersecting planes. The former is represented by [Figure 27](#) in the case of the roof (the plane KJLM) and the floor (the plane HIJK). KJLM and HIJK has one unified line, which is KJ. In other words,



the plane  $KJLM$  intersects with  $HIJK$  at the line segment  $KJ$ . This concept of two planes intersecting unperpendicularly is GeoGebra-illustrated in Figure 28.

**Table 3.** Positions of Planes

Position	Document	GeoGebra Illustration
Two Parallel Planes	 <p>(a) Without shading</p>  <p>(b) With shading</p>	 <p><b>Figure 26.</b> GeoGebra illustration two parallel planes</p>
Two Intersecting Planes Not Perpendicular	 <p>(c) Without shading</p>  <p>(d) With shading</p>	 <p>(a) Viewed from above</p>  <p>(b) Viewed from side</p> <p><b>Figure 28.</b> GeoGebra illustration two intersecting planes not perpendicularly</p>

Position	Document	GeoGebra Illustration
<p>Two Intersecting Planes Perpendicularly</p>	 <p>(a) Without shading</p> <p>(b) With shading</p> <p><b>Figure 29.</b> Two intersecting planes perpendicularly</p>	 <p><b>Figure 30.</b> GeoGebra illustration two intersecting planes perpendicularly</p>
<p>Two Planes Coinciding</p>	 <p>(a) With shading</p> <p>(b) With shading</p> <p><b>Figure 31.</b> Two planes coinciding</p>	 <p><b>Figure 32.</b> GeoGebra illustration two planes coinciding</p>

As for the concept of perpendicular intersecting planes, the illustration can be seen in [Figure 29](#) about the stairs of the mosque verandah. The concept can be found in the two components of the stairs, namely: the tread (the horizontal part to step on) and the riser (the vertical height to step up).



In [Figure 29](#), the plane NOPQ shows a tread and QPRS a riser. The two planes are rectangles. They intersect along the line QP perpendicularly ( $\text{NOPQ} \perp \text{QPRS}$ ). The resulting angles of perpendicular intersecting planes are right angles; therefore,  $\angle \text{NQS} \equiv \angle \text{SQN} \equiv \angle \text{OPR} \equiv \angle \text{PRO} = 90$ . [Figure 30](#) shows a GeoGebra illustration of this concept of perpendicular intersecting planes.

The last concept of intersecting lines found in the verandah of the mosque is that of coinciding planes. In [Figure 31](#), it can be seen decorative ornamental carving on the wall of the mosque. The plane ABCD represents the ornament and TUVW the wall. Based on observation and visualization, the plane ABCD coincides with that of TUVW. This is because the two planes have more than one unifying line, namely:  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{DA}$  lying on the plane ABCD and, simultaneously, TUVW. The GeoGebra illustration of this concept is shown in [Figure 32](#).

The learning activities that can be done can involve using the results of the analyses of the positions of points ([Table 1](#)), lines ([Table 2](#)), and planes ([Table 3](#)) during the apperception stage of the instructional process by using the Power-point slides. After learners understand the concepts of the standings of points, lines, and planes, through group work and discussion, the teacher can ask the students to find other examples of the positions of points, lines, and planes in objects they can find around the school. The results are documented and, then, presented in front of the class. In this case, learners will be able to have direct experiences in findings mathematical objects in their daily lives by looking at the teacher's examples of cultural sites so that learning will be more meaningful for them. Meanwhile, by presenting their work in front of the class, learners will acquire the knowledge and experiences so that they will grow more confident in presenting discussion results. This is because, through the presentation, they have the responsibility to present and make the class audience understand what they have to present.

## CONCLUSION

From the results of exploration by direct observation and identification through visualization processes aided by the GeoGebra software, a number of ideas about the application of the concepts of points, lines, and planes can be found. There are 16 mathematical concepts consisting of four concepts of the positions of points, eight of lines, and four of planes. The concepts of the standings of points include, among others, the position of points towards lines (points lying on lines and points not lying on lines) and the positions of points towards planes (points lying in planes and points lying outside planes).

Meanwhile, concerning the standings of lines, there are positions of lines against other lines (two parallel lines, two perpendicularly intersecting lines, two unperpendicularly intersecting lines, two intersecting lines, and two coinciding lines) and positions of lines against planes (lines parallel with planes, lines perpendicularly intersecting planes, and lines coincident with planes). And, the concepts of the positions of planes against other planes include two parallel planes, two unperpendicularly intersecting planes, two perpendicularly intersecting planes, and two coincident planes.

Geometrical mathematical objects have conceptually been implemented that can be seen in the architectural elements of mosque verandahs such as the one in the *Gedhe* Mosque of Kauman, Yogyakarta. Abstract visualization of the mathematical objects can be easily imagined through cultural sites, such as the great mosque in this study, to make it easier for the learners to understanding the instructional materials. Use of local cultures to be integrated in mathematics learning can challenge the students' curiosities and creativities so that they can acquire experiences that are meaningful, useful, and lasting long to be saved in their long-term memories. Visualizing the standings of points, lines, and planes enables to minimize extraneous cognitive loads so that it makes it easy for learners to understand and they will be challenged in independently constructing their understanding based on their pre-existing experiences.

Further research is expected to be done on identifying the mathematical concepts that exist in the architectural elements of buildings such as a mosque. Such studies can be directed to the elements of the the construction emphasizing on the kinds of the room forms and the kinds of angles that are formed. Subsequently, ideas can be proposed as to how the research results can be implemented in the school mathematics instruction.

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