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Effects of Gap on Erosion Surrounding Culvert Joints - An Experimental Study

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ABSTRACT

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Culvert is one of the drainage systems designed to transport water from one place to another place. Soil erosion can occur due to gaps in the joints of culverts, leading to phenomena such as piping, backward erosion, and suffusion caused by water discharge. When water flows through a gap culvert joint, it makes the soil around it fluid, which can result in sand boiling, and eventually, the formation of sinkholes. Water entrains soil particles, reducing soil density around the culvert, forming voids that expand with erosion. A laboratory-based study investigated gaps in culvert joints, and found that water flow velocity and duration significantly influence soil erosion. Water flow transports eroded materials into culverts, with most soil particles settling around the joints. The gaps of culvert joints can impact erosion, with larger gaps leading to greater influx of soil particles into the culvert. Longer flow durations also result in greater erosion, as fluidization of the soil leads to increased erosion. On the other hand, larger lengths of culvert joints result in reduced erosion. Soil accumulation along the culvert joints cover and clogging contribute to this phenomenon. Culverts lacking joints cover suffer significant erosion, with soil particles entering the culvert. Sedimentary material is predominantly found within the culvert rather than being carried out by the flow. Additionally, the size of eroded soil particles affects the width of the gaps, typically ranging from 0.25 - 0.4 mm. The erosion phenomenon that occurs around culvert joints can lead to long-term effects.

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1. Introduction

Culverts serve as a vital component in the management of water flow, drainage facilitation, and flood prevention in field applications. The installation of culverts typically follows a segmented approach to ensure optimal performance. This method process provides alignment precision and effective water management. However, culvert joints as shown in Figure 1 may develop gaps as a result of the segmented installation process, leading to soil erosion and other complications when water flows through. It is important to note the potential consequences of such gaps and address them accordingly to maintain the integrity of the culvert system.

Sinkholes occur on roads or sidewalks, typically caused by the load imposed by traffic flow. The majority of drainage culverts are unclassified pipes, wherein wastewater and rainwater flow together [1]. Gaps at culvert joints, as depicted in Figure 2, typically occur at joint areas due to inaccurate in culvert joint installation, material properties, traffic loads, soil deformation, or the aging of culverts. Logistic regression analysis on most of drainage culvert pipe data in metropolitan areas, considering various factors including length, height, age, equivalent radius, slope, burial depth, and pipe material. However, this study did not consider the geological and geotechnical properties of the soil, which are crucial factors related to soil infiltration through damaged pipes, cavity evolution, and underground instability [1].

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(b) **Figure 1.** Precast concrete conduit: (a) schematic (b) installation [2]

Figure 2. Sinkhole occurrences due to defective sewers [1]

Sinkhole phenomenon and ground subsidence pose dangers due to the difficulty of predicting such occurrences [3] [4]. Wireless sensor networking (WSN) methods are developed to monitor leaks in underground pipes and sinkholes that occur. Although further investigation and extensive research are still needed [5]. A numerical simulation of underground structure failure, water flow, deformation, and internal soil erosion was conducted. The results indicate that underground cavities form due to water flow from leaking conduit [6]. Therefore, water pressure and hydraulic gradients increase at the leakage source. Interaction between soil particles and water flow causes internal fluidization, resulting in significant soil deformation [7]. Deformation of the soil leads to underground cavities and surface soil collapse [8].

Soil erosion occurs with increasing hydraulic gradients, causing sand boiling [9] [10]. Eroded particles are predominantly small-sized [11]. Material transport ensues rapidly and continuously post-erosion [12]. A higher hydraulic gradient is correlated with increased mass of eroded clay [13]. Soils with a uniformity coefficient of 1,5 necessitate a higher hydraulic gradient for initiating backward erosion [14]. The erosion rate is proportional to seepage velocity $[15] [16] [17]$.

Soil erosion displaces fine particles due to seepage flow through gaps between larger particles, leading to erosion and subsequent expulsion of fine particles, known as suffusion. This process involves a spreading phenomenon accompanied by clogging-induced excessive pressure, culminating in blowouts [18] [19]. Observations of cyclic infiltration and exfiltration flow reveal three erosion behaviors: clogging, incomplete erosion, and complete erosion [20]. The erosion process comprises five distinct stages: indistinct infiltration failure, erosion initiation, erosion development, erosion intensification, and comprehensive failure [21].

Sinkholes from drainage conduit leaks are complex and challenging to predict [22] [23]. Several researchers have researched drainage conduit leaks. Larger leakage holes in the drainage conduit make greater evolution of erosion. Sinkholes formed are cone-shaped and located directly above the drainage conduit leakage site [21]. Infiltration into drainage pipe leaks causes soil material to enter defects in the drainage pipe [24] [25]. Coarse and medium-sized non-cohesive materials are found to be easily eroded [26]. As eroded particles flow and disappear, carried by the water flow, the shear of the eroded sand particles forms erosion zones that expand over time [22] [27]. Soils with high fine content will cause unstable arches to form and allow soil particles to enter through gaps in drainage channel leaks [20]. Uniformly graded soils with no fine content will experience significantly greater erosion than well-graded soils with fine content. Loose soils will increase the intensity of erosion and soil deformation [28] [29]. Continuous exfiltration and infiltration flow through gaps in drainage conduits can increase erosion. Soil erosion due to drainage conduit leaks was controlled by two factors: water flow rate and particle size ratio [20] [30].

The vulnerability of drainage pipe leakage can lead to severe socio-economic losses. Erosion is a complex [22] and recurrent process [12]. Research on the erosion mechanisms caused by gaps in culvert joints has yet to be conducted before. Sinkholes formed from erosion at culvert joints are challenging to detect. This study aims to investigate the effects of gap width at culvert joints, considering flow duration and culvert joint cover length. By studying the mechanism of erosion, expected to provide insights into the mechanisms of erosion caused gaps of culvert joints.

2. Methods

2.1Experimental Setup

The experimental setup, illustrated in Figure 3 and Figure 4, simulates soil erosion around culvert joints. It consists of a soil box measuring 30 cm wide, 92 cm long, and 44 cm high. Two culverts with a diameter of 15 cm, constructed from 5 mm thick acrylic material, are utilized in the experiment. In this experiment, we varied the gaps at the culvert joints by 2 mm, 4 mm, and 6 mm to analyze their effect on the erosion mechanism.

Incoming water was measured at the inlet section using a flowmeter. Subsequently, soil samples are inserted and compacted within the sandbox. At the outlet section, outgoing water is filtered using a No. 200 sieve to collect the eroded material. The entire procedure is conducted with meticulous attention to detail and precision to ensure the acquisition of accurate and relevant data.

Figure 3. Experimental system for soil erosion (not to scale, units in millimeter)

Figure 4. Experimental set up

2.2Tested Material

Sand from the Progo River was used in this study, with its parameters listed in Table 1. The soil samples comprise natural sand, with particles retained No. 200 sieve. Based on the results of particle size analysis as seen in Figure 5, the soil is classified as uniformly graded, dominated by particles sized between 0.25 - 0.43 mm.

Table 1. Soil properties of Sungai Progo sand

Classification in USCS	SP (Sand Poorly Graded)
Specific Gravity (G_s)	2.68
Mean Grain Size D_{50} (mm)	0.41
Coefficient of uniformity (C_u)	0.71
Coefficient of gradation (C_c)	0.26
Permeability (cm/s)	0 03

Figure 5. Particle size distributions

2.3Experimental Procedures

The research process begins with the careful preparation of experimental equipment, as depicted in Figure 3. Soil was compacted into layer by layers with a porosity of 0.4, simulating the condition of loose granular soil, commonly found in poor construction conditions [20].

Water was conveyed into the culverts with a constant head, and the flow rate was precisely measured with a flowmeter. The gap variations are conducted at 1, 2, 4, and 6 mm to observe the effect of gap width on soil erosion. The width of gaps in the culvert joints is referred to as *B*. Variations in flow time (*T*) are also conducted with each variation in gaps at the joints. Furthermore, variations in the influence of using culvert joints length (*L*) are conducted with length of 5, 3, and 1 cm.

The sedimented material's weight and the weight of the material retained on the number 200 sieve are then carefully weighed and recorded. This experiment was conducted to observe the effects of various parameters on soil erosion.

3. Results and Discussions

3.1 Observation of Erosion

Soil particles are mixed with water due to seepage at culvert joints. Fluidization is the phenomenon of a solid material changing its properties to become fluid-like due to contact with a liquid or gas. Soil particles unable to bear their own weight are carried by the water flow. Following water discharge from the culverts, exfiltration and infiltration occur through the gaps in the culvert joints [20]. Water becomes turbid as soil particles enter the culverts. Upon completion of the testing, erosion leads to the formation of sinkholes, lowering of the ground surface, and sedimentation within the culverts. With the loss of sand particles, an erosion zone forms on the sand surface.

Water flow carries soil particles, resulting in infiltration and exfiltration [20]. The formation of particle contacts during compaction prevents soil loss. Seepage can lead to soil fluidization around culvert joint gaps. In fine particle infiltration through culvert joint gaps, sinkhole size increases with larger culvert joint gaps (*B*) and longer flow duration (*T*). The resulting sinkholes resemble an inverted cone and are located centrally above the culvert joint gaps. Similar occurrences have been reported in studies conducted by [20] [28] [22]. Infiltration and exfiltration processes lead to sudden collapses as shear strength can no longer support soil weight above internal voids.

3.2 Effects of Gaps on Eroded Material

This study was conducted to explore the variations in the gap width of culvert joints (B) . As represented in Figure 6, the gap widths in culvert joints were experimentally varied to 1 mm, 2 mm, 4 mm, and 6 mm. The outflow of water from these gaps resulted in erosion, which was studied in relation to the gap width and the corresponding amount of eroded material.

The correlation between the gap width of culvert joints (*B*) and the eroded material (*W*) at culvert joint cover (L) $= 50$ mm as seen in Figure 7. Results of this study are presented in Table 2, which illustrates the variations in erosion corresponding to different gap widths (*B*) of culvert joints.

Figure 6. Variation gaps in culvert joint

Figure 7. Effects of gaps on eroded materials with culvert joint cover $(L) = 50$ mm

Table 2. Results of culvert joint gap variations on 50 mm culvert joint cover

No.	Gap width of culvert joints, B (mm)	Flow duration, T (minute)	Eroded materials, W(g)	Increase in erosion $(\%)$
1	1	2	29.74	
	2	2	43.63	46.70
	4	2	92.64	112.33
	6	2	117.14	26.45
2	1	4	37.15	
	2	4	64.23	72.89
	4	$\overline{4}$	125.14	94.83
	6	4	133.63	6.78
3	1	6	66.43	
	2	6	71.06	6.97
	4	6	157.42	121.53
	6	6	160.36	1.87

The results presented in Table 2 indicates that there was a direct correlation between gaps width of culvert joints (*B*) and an increase in material erosion. The most significant erosion of material occurs with a gaps width of culvert joints (*B*) of 6 mm and a flow duration (*T*) of 6 minutes, reaching 160.36 grams. The amount of eroded material increases with the widening of gaps at culvert joints.

Testing was also conducted on culvert joints cover (*L*) 30 mm. Correlation between the gap width of culvert joints (*B*) and the eroded material (*W*) at culvert joint cover (L) = 50 mm as seen in Figure 8. Results of testing various gap widths with culvert joints cover (*L*) of 30 mm are shown in Table 3.

Figure 8. Effects of gaps on eroded materials with culvert joint cover $(L) = 30$ mm.

Refer to Table 3, the highest erosion occurs at a gap width of culvert joints (*B*) of 4 mm with a flow duration (*T*) of 6 minutes, amounting to 290.22 grams. The most significant increase in erosion is observed in the test with $B = 2$ mm and $T = 4$ minutes, which resulted in 114.38 grams compared to $B = 1$ mm and $T = 4$ mm. From each test with the same flow duration (*T*), the eroded material increases with the widening of the gap width (*B*) at the culvert joint.

Variations in gaps of the culvert joint were also conducted on the culvert joint cover of 10 mm. The correlation between the gap width of culvert joints (*B*) and the eroded material (W) at culvert joint cover (L) = 50 mm, as seen in Figure 9. The results of this variation are presented in Table 4.

Figure 9. Effect of gaps on eroded materials with culvert joint cover $(L) = 10$ mm

Table 4. Results of culvert joint gap variations on 10 mm culvert joint cover

No.	Gap width of culvert joints, B (mm)	Flow duration T (minute)	Eroded materials W(g)	Increase in erosion (%)
1	1	\overline{c}	106.78	
	2	\overline{c}	352.36	229.99
	4	\overline{c}	409.82	16.31
2	1	4	119.93	
	2	4	392.8	227.52
	4	4	469.02	19.40
3	1	6	207.02	
	2	6	401.35	93.87
	4	6	585.87	45.97

The test results in Table 4 indicates that the maximum amount of erosion, amounting 585.87 grams, occurs when the gap width of culvert joints (*B*) is 4mm and the flow duration (*T*) is 6 minutes. On the other hand, the most substantial increase in erosion, which amounts to 245.58 grams, is observed when (B) is 2mm and (T) is 2 minutes.

Based on the test results that the eroded material (*W*) has a linear correlation approach with the gap width at the culvert joint of culvert joints (*B*). In addition, in the tests conducted with culvert joint covers (*L*) of 50, 30, and 10 mm, the maximum erosion occurs when (*B*) is 6 mm. Increasing (*B*) results in an increase in (*W*) for the same flow duration. Figure 7, Figure 8, and Figure 9 illustrates the correlation between the influence of (*B*) on (*W*) for each culvert joint cover.

Figure 9 shows a direct correlation between *B* and *W*. The eroded material with $W = 1$ mm experiences less erosion. The erosion processes result in the formation of inverted cone-shaped holes due to soil collapse [22] [24] [28]. Soil is transported by water flow in narrow zones over culvert joint gaps. Numerical tests reveal the loss of soil particles near the channel leakage due to gravitational and tensile forces, while inter-particle friction had a negligible effect as per [24].

3.1 Effects of Flow Duration

This study investigates the influence of flow duration (*T*) on erosion. Specifically, the research focuses on the susceptibility of fluidized soil to erosion, which can lead to sediment accumulation within culverts. This sedimentation, in turn, forms soil mounds above the culvert joint gaps. The correlation between flow duration and the amount of eroded soil was examined, and the findings are presented in Figure 10.

In Figure 10, an increase in the weight of material carried by water flow was observed to occur with prolonged flow duration, due to the extended period of soil fluidization. The continuous presence of water flow was noted to cause erosion, with soil particles mixing with water and entering the culverts. The eroded material measured between 0.25 - 0.4 mm.

Figure 10. Effects of water flow on eroded materials for culvert joint cover (*L*): (a) 50 mm; (b) 30 mm, and (c) 10 mm

3.2Effects of Culvert Joint Length (*L***)**

A model was developed to determine the length of culvert joints, which is characterized as *L* (mm). The study considered different lengths of *L*, including 1, 3, and 5 cm, as well as not using any joints, as seen in Figure 6. The correlation between *L* and *W* is illustrated in Figure 11. This information could be valuable to stakeholders and external audiences who need to make informed decisions about culvert design.

From the research results, the culvert joint cover (*L)* affects the amount of eroded material. Figure 11 illustrates that as *L* increases, *W* also increases. It is evident from Figure 12 that in all tests, as L increases, the amount of eroded material also increases. The smallest value of *W* is observed when $L = 5$ cm due to clogging, as seen in Figure 12. Longer culvert joints result in more soil in the clogging zone. A larger *L* allows for more material to be accommodated in the clogging zone, thereby reducing the amount of material entering the culverts.

Figure 11. Effects of culvert joints length (*L*) on eroded materials (*W*)

(b)

(c)

Figure 12. Soil that fills along the culvert joint cover (*L*): (a) 50 mm, (b) 30 mm, and (c) 10 mm

4. Conclusions

This study used a simple experimental model to simulate erosion occurring at gaps of culvert joints, using sand as the erosion-vulnerable material. Maintaining a consistent water flow rate, the study observed the impact of flow duration on gap width at culvert joint openings. The greater gap width of culvert joints (*B*) corresponds to larger values of material eroded (*W*). Soil erosion in culverts can be intensified by wider gaps in the joints, allowing more soil to enter and accumulate within the culverts. Greater flow duration (*T*) corresponds to larger values of material eroded (*W*). Longer flow durations can intensify soil erosion, and long-term fluidization of eroded material can further exacerbate erosion. Increasing culvert joint cover (*L*) results in smaller values of material eroded (*W*). Culvert joints with a length of 5 cm result in less soil erosion compared to those with lengths of 3 cm and 1 cm, as soil particles experience blockage and get trapped along the joint length. The majority of sedimentation material is found inside culverts rather than being carried away by the flow of water. The eroded material is predominantly composed of particles ranging from 0.25 to 0.4 mm in size.

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