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Didactical design of algebraic expression from a linear pattern with a realistic mathematics education approach

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ABSTRACT

The design of didactic mathematics learning is designed by taking into account the learning barriers of students. This study aims to obtain a didactic design for learning algebraic expression from a linear pattern to reduce student learning barriers through the application of Realistic Mathematics Education (RME). The type of this research is design research with three stages, namely the plan, do, and see. The research data was in the form of qualitative data with the research instruments used were Hypothetical Learning Trajectory (HLT) on algebraic forms of linear patterns, student worksheet, pretest and posttest, observation sheets, teacher interview sheets, and student interview sheets which were then analyzed retrospectively. This research constructs a didactic design in the form of a learning trajectory divided into nine stages and packaged in four activities. The applied didactic design opens the thinking process for students to find the meaning of variables and algebraic forms of the mathematical activities themselves. This didactical design is proven to reduce students' learning obstacles.

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INTRODUCTION

Mathematics is one of the subjects that students will study at every level (Astuti & Wijaya, 2020, p.2; Confrey et al.,2014). Mathematics taught at each level have a hierarchical relationship, which means that students' understanding of mathematics topic at the previous level will become an asset for students to understand further mathematics topics. Therefore, teachers must pay full attention to all aspects in the mathematics learning process so that all learning objectives can be achieved optimally at every level. In the learning process, there are at least three main components in the didactical triangle, namely teachers, students, and content (Kansanen, 2003, p. 229; Wilson, 2013). Each of these three components will form a relationship that must be viewed as an integral part to achieve optimal learning objectives. The relationships between these components will later create a didactic situation when learning process occur. If a didactic situation has been created, then this situation will be able to form a learning process in the minds of students including physical activity and mental activity.

Based on the education curriculum in Indonesia, one of the mathematics topics that must be taught is algebra. Permendikbud Number 7 of 2022 concerning content standards states an introduction to algebra is taught in grade 8. Algebra is considered difficult by students at school. Most students having difficulties in mathematics are affected by the failure to understand the introduction of algebra taught in the first year of junior high school. It is in line with the results of several previous studies, where the researchers stated that students find mathematics difficult when they enter algebraic topics. In learning algebra, students need to think abstractly and have many new skills in solving problems, including symbol representation, equations, graphs, and logical reasoning skills (Stewart et al., 2019, p.8-9). At the elementary level, the mathematics taught is still too elementary and concrete. However, at the junior high school level, students find that the topic of introduction to algebra is very difficult. It is because most teachers teach algebra by directly giving examples of algebraic forms without explanation about what each element in the algebraic form means. Consequently, it becomes a factor of learning obstacle for students in understanding algebra (Ardiansari, 2019, p.32).

Algebra, as indicated by Stewart et al. (2019), marks a pivotal point in mathematical education where abstract thinking becomes essential. This stage demands proficiency in symbol manipulation, equation solving, graph interpretation, and logical reasoning. However, Ferretti (2019) highlights the prevalent struggle among students in mastering algebraic concepts. Despite efforts, correct responses remain scarce due to challenges in algebraic manipulations. Recognizing this hurdle, Dibbs et al. (2020) delve into effective teaching methods to alleviate these learning barriers. Their findings endorse small-group instruction as an effective strategy to bolster students' grasp of algebra. Echoing these sentiments, Hord and Hoyng (2021) stress the significance of diverse instructional approaches in secondary education. Their research underscores common obstacles encountered by students, such as organizational difficulties and lapses in procedural memory. Innovative strategies like the box method, arrow usage, and combining like terms prove instrumental in overcoming these challenges. Thus, these studies collectively underscore the importance of tailored pedagogical approaches to enhance algebraic comprehension and mitigate student struggles in transitioning from arithmetic to algebra (Watt et.el., 2014).

Several studies have been conducted by researchers regarding didactic design in reducing student learning barriers on several mathematical topics. For instance, research by Pratamawati (2020) regarding designing didactic designs to overcome student learning barriers in high school-level material. She found that learning barriers were experienced by students with various levels of academic ability. In addition, research conducted by Putri et al. (2020) regarding a didactic design to reduce student learning barriers in the topic of matrices shows that there were ten learning barriers experienced by students, including an inability to identify the form of a matrix and non-matrix, difficulty in determining between rows and columns in the matrix, difficulty in determining the location of matrix elements, and several other learning barriers. Astriani et al., (2022) conducted a study on the preparation of didactic designs to overcome barriers to learning mathematics for junior high school students. The research was conducted in class VIII on the concept of flat-sided geometric shapes and was designed based on the didactic situation and the students' didactic obstacles that had been previously identified. The results of the retrospective analysis carried out succeeded in producing a didactic design that was able to reduce student learning barriers in mathematics material. Then, previous research on LT in algebra conducted by Gürbüz & Ozdemir (2020) found four points in LT to prohibit students from meaning variables, namely students using literal expressions that are familiar to students in understanding algebraic forms, students can notice patterns and express them in the form of variables, students can write algebraic expressions and substitute them with numbers, and students can discover the concept of variables as something they do not know. Furthermore, Santri et al. (2019) conducted a study that aimed to produce a learning trajectory using mathematical modeling to help students understand the concept of algebraic operations in junior high school. The research invented LT algebraic operations with three activities, namely understanding algebraic expressions, doing algebraic addition, and doing algebraic absorption.

In several research, it appears that there is a lot of research on designing didactic designs for specific materials in mathematics. Two of these studies show the results of producing LT in algebraic material, especially regarding the meaning of variables and algebraic operations. Many results of the research conducted also show effectiveness in reducing student learning barriers. The resulting LT is local in terms of material and research subjects. Therefore, researchers are interested and need to conduct similar research but differ in terms of material and research subjects. The research carried out is about the didactical design of algebraic forms of linear patterns because several studies have shown that students have great difficulty understanding algebraic learning related to variables. Researchers aim to help students discover the concept of variables through observing patterns. All activities are designed with enacting learning that can bridge concrete arithmetic transitions to abstract algebra.

To overcome the learning barriers experienced by students in learning algebra, there are several things that must be done, namely designing a didactic mathematical design using a learning approach that is in accordance with the characteristics of algebra that is a didactic mathematical design based on learning barriers experienced by students and contains Hypothetical Learning Trajectory (HLT) which is suspected to be appropriate and effective. To make algebra more concrete for students, the choice of learning approach applied in teaching algebra must be appropriate. The learning approach that is considered efficient for teaching algebra is a learning approach that contains the characteristics of math as a human activity, which means that students will gain an understanding of algebraic concepts through mathematical activities (Freudental, 2002, p. 14). One approach to learning mathematics that contains these characteristics is Realistic Mathematical Education (RME).

RME is a learning approach specifically designed for learning mathematics and contains several characteristics, namely (1) the use of context, (2) the use of emergence models, (3) student contribution, (4) interactivity, and (5) intertwinement. Based on these characteristics, it can facilitate students' understanding of the introduction of algebraic concepts. Contextual problems that are used as starting points in learning algebra are contexts that are familiar to students' minds, and can provide opportunities for students to find various kinds of models in bridging students' thinking to arrive at a correct understanding of algebraic concepts through mathematical activities designed by the teacher, and can support the creation of didactical situation and the occurrence of meaningful learning in the process of learning mathematics on the topic of algebra being carried out.

Based on this description, researchers carried out research on designing didactic designs of algebraic forms from a linear pattern with a Realistic Mathematics Education approach. The research was conducted at one of the State Junior High Schools in Samarinda. The school was chosen as the research location since the students' mathematics scores on the topic of algebra were still relatively low and based on interviews with the mathematics teacher at the school, the teacher had never applied the RME approach in teaching algebraic topics. Learning about algebra was also carried out dominantly only based on existing textbooks. This raised suspicions in the minds of researchers, that the learning trajectory applied was not appropriate and effective in learning algebra at school. So that the learning process still created learning obstacles for students which implies learning objectives are not achieved optimally. The learning barriers experienced by students must be further identified, whether due to students' lack of mental readiness to learn (ontogenic obstacle), the wrong choice of learning methods and approaches (didactic obstacle), or the limited ability of students to apply mathematical concepts in solving a problem (epistemological obstacle) (Brousseau, 1997, p. 83). It is because the making of the didactic design will be oriented toward the learning obstacles experienced by students. The research question in this study is what are the characteristics of the didactic design that can reduce students' learning difficulties in learning the algebraic form of a linear pattern? Learning obstacles in this study are students' obstacles in learning about algebraic expressions.

METHODS

The type of this research is design research that is applied to create a mathematical didactic design packaged with the RME approach (Plomp & Nieveen, 2013, p.17). The product of this study is a didactic mathematical design and learning trajectory for learning algebraic expressions of linear patterns. This research involved three stages, and these three stages were carried out in two cycles, namely pilot experiments and teaching experiments (Suryadi, 2019, p.22). The three stages were; (i) plan, where in this stage the researcher conducted a prospective analysis, namely examining relevant research results, analyzing and observing the characteristics of the algebraic material to be taught, examining contexts that can be used in teaching algebra, conducting interviews with mathematics teachers at school, designing a Hypothetical Learning Trajectory (HLT) for learning algebra, and preparing an initial didactic design; (ii) do, where in this stage, the researcher implemented a mathematical didactical design along with the HLT made in learning algebraic forms from linear patterns and carried out a metapedadictic analysis that is flexible, unified, and coherent, and; (iii) see, where in this stage, the researcher conducted a critical, reflective analysis of the implementation of the mathematical didactic designs that had been made along with the HLT for learning algebraic forms from linear patterns. In addition, the researcher considered several things in the third stage, namely, the accuracy of the pedagogical didactic actions given, the accuracy of predicting student responses, the accuracy of anticipation of student responses, the accuracy of HLT in algebra learning in grade 8th, and whether mathematical didactic designs and HLT made for algebra learning needed to be carried out revision or not. When there was a need for revisions, the researcher revised and re-applied the revised didactic mathematical and HLT designs to teaching

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experiments in order to obtain effective didactic mathematical and LT designs in teaching algebraic expressions of linear patterns to students.

The research was conducted at a public junior high school in the city of Samarinda, East Kalimantan, from 19 September 2022 to 19 November 2022. The subjects in this study were grade 8th students. The technique for taking subjects in this study was purposive sampling, namely determining research subjects with several criteria (Creswell, 2012, p.204). Some of the criteria for determining the subject of this study were students who had learned fundamental arithmetic competencies and students with high, medium, and low abilities. The subjects selected in the pilot experiment (cycle I) numbered 6 to 8, while the subjects selected in the teaching experiment (cycle II) were 30 students.

Data collection techniques used included teaching observations applied by mathematics teachers, observations during the learning process in pilot experiments and teaching experiments, observations of students' learning obstacles from the results of pretest and posttest cycle I and cycle II, interviews with mathematics teachers and interviews with students who became research subjects after participating in the learning process, pretest and posttest were carried out on pilot experiments and teaching experiments with different research subjects, pilot experiments namely class trials were carried out to test initial didactic designs and HLT algebra learning in class VII, and teaching experiments were carried out based on design results didactic mathematics and revised HLT in the pilot experiment.

The research instruments used were i) Hypothetical Learning Trajectory (HLT) algebraic expressions of a pattern, ii) instructional tasks packaged in worksheets with the RME approach, iii) observation sheets, iv) interview sheets, and v) pretest question papers and posttest questions. The validity of the data, namely the validity of the research data, was carried out by data triangulation, namely collecting video recordings of research activities and student worksheets (Cohen et al., 2018, p.133). The reliability of this research data was analyzed by conducting an audit of the entire process of research activities through recordings, photos, student worksheets, and other important records that had been collected.

Analysis of the research data is a retrospective analysis, namely an analysis that is critical and reflective of all stages of the research conducted. A retrospective analysis of the results and findings in this study was carried out through intensive discussions with the research team. Data from teacher interviews were analyzed by looking at the answers given as an illustration for researchers of previous algebra learning situations. The results of student interviews were analyzed by looking at student answers as an illustration for researchers about student learning obstacles during activities and the accuracy of the learning trajectory applied. Data from observations of teachers were analyzed by looking at the learning activities carried out by the teacher and the situation and class conditions that occurred during learning. Observation data on students' work results were analyzed by looking at the flow of thinking and students' mathematical methods, as well as the difficulties identified from the location of the errors written on the worksheet. Pretest results data were analyzed by looking at the strategies and student answers on the answer sheet. The results of this analysis were used to identify students' learning obstacles before applying the didactic mathematical design of algebra learning in class VII. Data from pilot experiments and teaching experiments were analyzed using retrospective analysis. The focus of the analysis was learning obstacles during learning, the accuracy of the HLT applied, and the accuracy of the didactical/pedagogical actions carried out by the teacher. Posttest results data were analyzed by looking at the strategies and students' answers on the answer sheet. The results of the analysis were used to see students' learning obstacles and increase students' understanding of algebraic concepts after applying a mathematical didactic design.

RESULT AND DISCUSSION

The results and discussion in this article are used to answer research questions, namely about how the characteristics of didactic designs are effective in reducing students' learning obstacles in learning algebraic forms from linear patterns. In cycle I and cycle II, three stages were carried out, namely pretesting to identify students' learning obstacles in learning algebra, especially about linear patterns and algebraic forms of a pattern, then applying HLT to pilot experiments and teaching experiments, and giving posttests to see the effectiveness of the design. The HLT that has been implemented is indicated by the reduced learning obstacles of students.

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The didactic design is designed by making HLT along with a description of the activities to be carried out and instructional tasks to assist students in achieving the desired learning goals. HLT contains the main competencies to be achieved, namely students' understanding of the concept of linear patterns and the algebraic expressions of a pattern. To achieve these competencies, there are nine stages of learning that must be passed by students. These nine learning stages are conducted in 4 activities. In addition, these instructional tasks are also arranged in a student worksheet which is carried out together with the series of activities that have been designed.

In activity 1, students worked with matchsticks. Students arranged matchsticks into an arrangement of the letter M that has a pattern. Students observed the pattern in the arrangement of letters formed. The teacher directed the student's focus on the similarities, differences, and changes in each sequence of arrangements that are formed as shown in Figure 1.

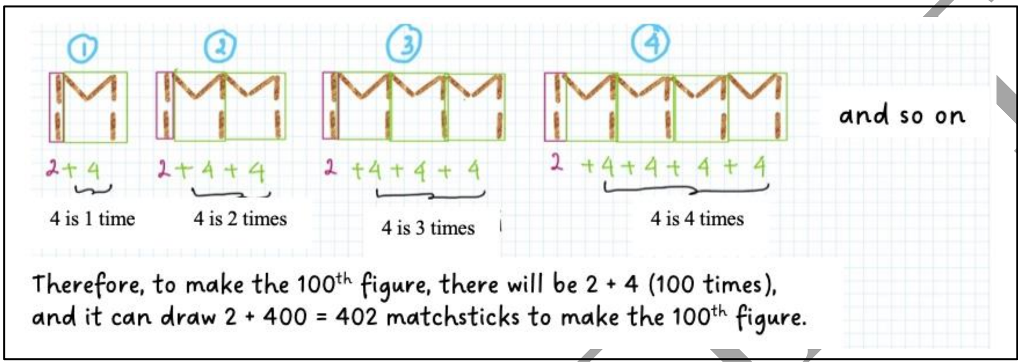

Figure 1. Pattern prediction on the arrangement of the letter M

Based on Figure 1, with the guidance given by the teacher in the form of coloring on the arrangement of letters formed, students can easily observe the changes that occur from each arrangement of the letters. In addition, students write their representation in repeated addition form and then convert it into multiplication form. Thus, after obtaining the representation of the pattern in the form of multiplication, students can easily generalize the pattern for the 100th order. From the experiences students have in Activity 1, students understand how to recognize, predict, and generalize patterns from the arrangement of objects. Student understanding in Activity 1 can help students understand the concept of patterns in the arrangement of numbers designed in Activity 2.

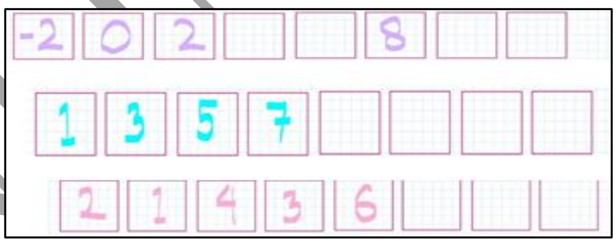

Figure 2. Number Card

In activity 2, students worked with number cards. Students were given a packet of number cards. Then students were asked to observe the pattern on the number card and predict the right number to fill in the blank number card, as shown in Figure 2.

In this activity, students can determine the correct number on an empty number card. However, students have difficulty in writing representations of pattern observations made on the arrangement of numbers. This shows that students experience learning obstacles in the form of an inability to apply the concept of patterns to the arrangement of objects that have been obtained in Activity 1. In addition, students also have difficulties in applying the multiplication concept in the form of repeated addition which is done to obtain a generalized pattern. Based on the learning obstacles that occurred in activity 2, researchers made improvements to the HLT which were designed to be implemented in cycle II. The HLT improvements made aim to reduce student learning obstacles in cycle II. The improvement made is adding guided worksheets to assist students in writing representations of observed patterns in the number arrangement. Guided added as shown in Figure 3.

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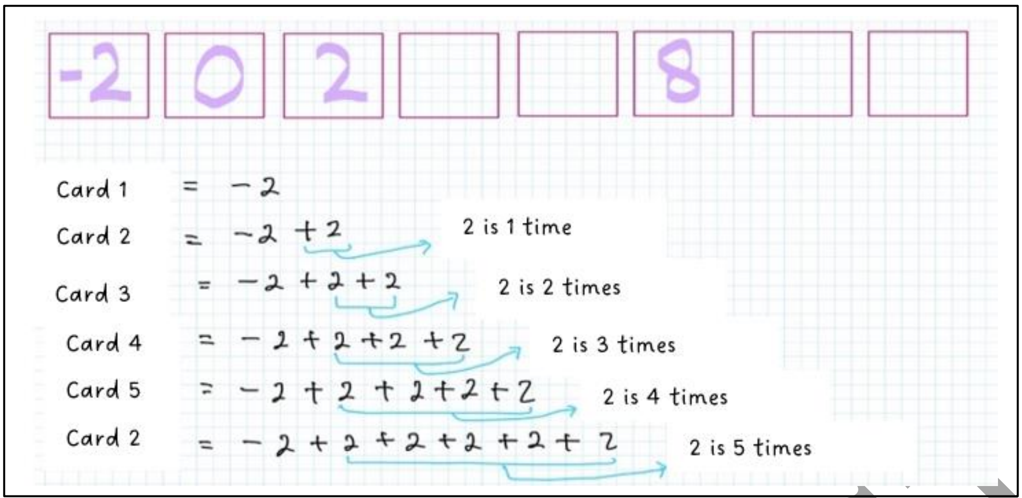

Figure 3. Additional *Guidance* in Activity 2

With the additional guidance given, students can easily make representations until they find a generalization for the 100th pattern of the number arrangement. The difficulties experienced by students in activity II are due to the fact that working with numbers felt more abstract in the minds of students than working with objects. This is in line with the results of relevant research that has been done previously, that student learning obstacles that often occur in learning algebra are caused by algebra having abstract characteristics and a transition from arithmetic to algebra (Dibbs et al., 2020, p. 219; Ardiansari, 2019, p. 32; Stewart et al., 2019, p. 8-9). Therefore, a revision of the HLT that is applied is really needed to help students go through the transition from concrete to abstract, especially in activity 2. After the objectives of Activity 1 and Activity 2 are well achieved, students can easily carry out Activity 3.

In activity 3, namely student production patterns, students arranged matchsticks into an equilateral triangle shape. Then students independently found the generalization of the 100th pattern from the arrangement made. The purpose of this activity 3 was to help students make generalizations of patterns in algebraic form so that when students succeed in finding the generalization of the 100th pattern, their focus was directed to the numbers that were always fixed and the numbers that are always changing from the written representation. Then, students were asked to replace numbers that were always changing with a symbol which in formal mathematics is called a variable. After this, students were asked to explain the meaning of the variable from the resulting algebraic form and relate it to the context being worked on. In activity 4, students were given contextual problems and students were able to solve them by applying the concept of algebraic forms from linear patterns obtained from a series of previous activities. Figure 4 presents the results of student work in solving contextual problems.

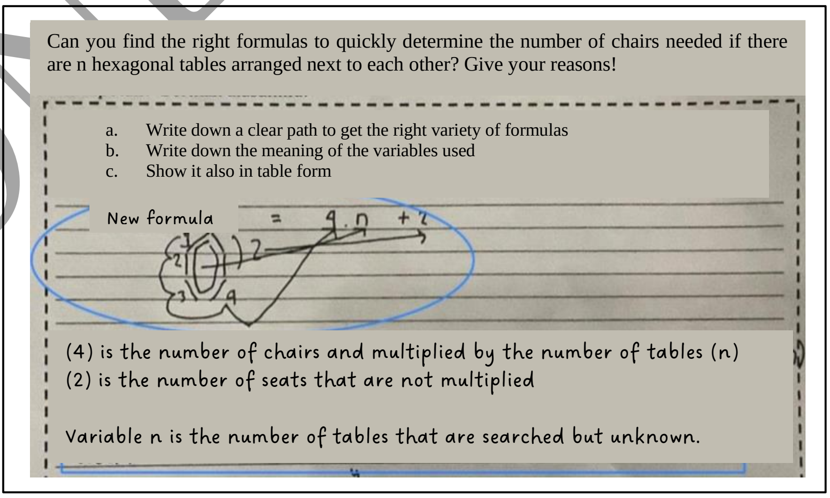

Figure 4. One of the results of student work on the Contextual Problem

From the series of activities carried out, it can be seen that students can follow the entire series of activities well, especially after revisions to the HLT are applied. In addition, students can go through all stages of learning and achieve learning objectives from each activity carried out. Thus, students get a complete understanding of the concept of algebraic forms of linear patterns. This shows that the resulting learning trajectory is appropriate in teaching algebraic forms from linear patterns and is effective in reducing students' learning obstacles in the learning being carried out. The resulting learning trajectory is shown in Figure 5.

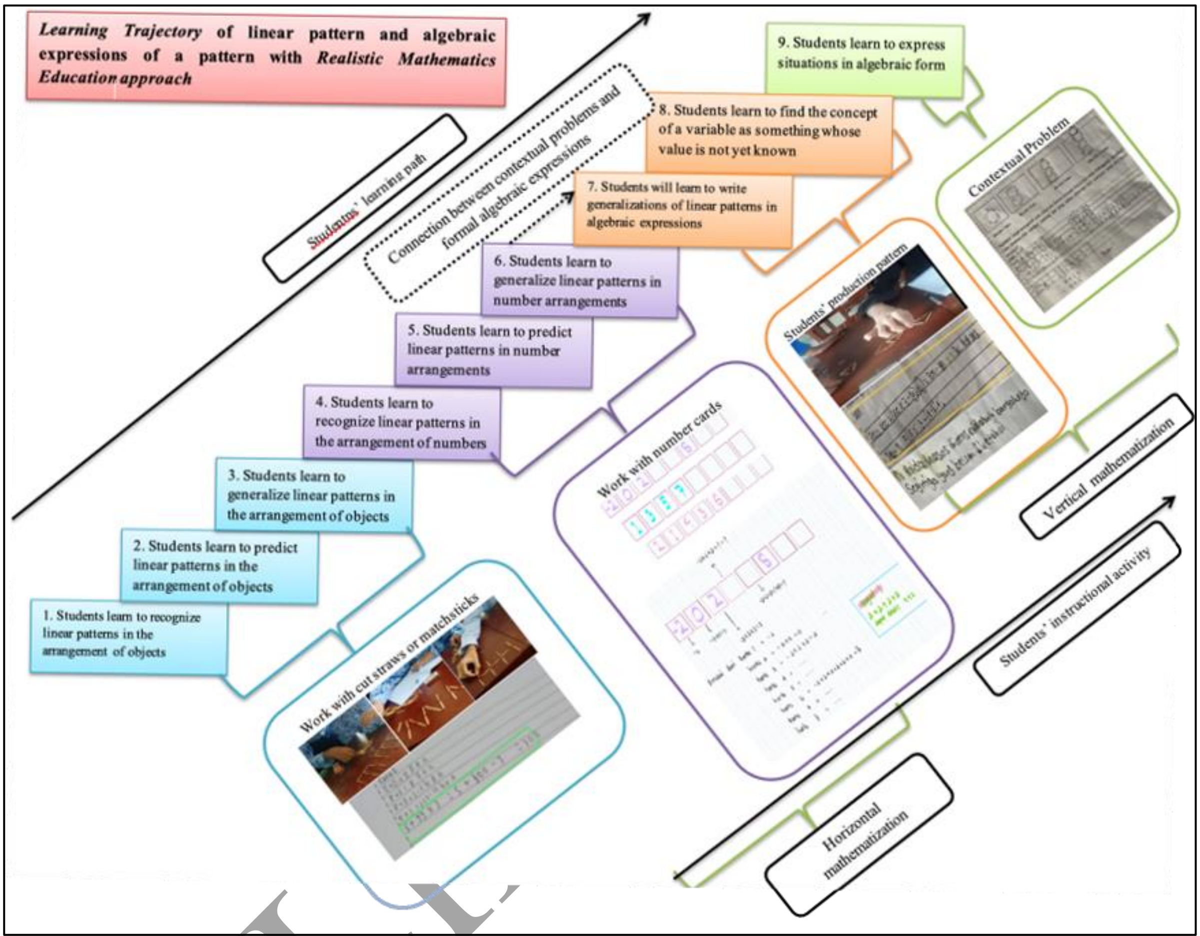

Figure 5. Local Learning Trajectory on Linear Patterns and Algebraic Expression of a Pattern

In addition, to see the effectiveness of the applied learning trajectory, the results of the students' pretest and posttest scores are compared. In the pretest and posttest students were given 5 essay questions with a level of difficulty of 2 easy questions, 1 medium question, and 2 difficult questions. Question number 1 is to find out students' ability to recognize, predict, and generalize patterns in object arrangements, question number 2 is to find out students' abilities to recognize, predict, and generalize patterns in number arrangements, question number 3 is to find out students' abilities to generalize algebraic expression of a pattern, question number 4 is to find out students' ability to explain the meaning of variables from the algebraic expression associated with the context being presented, and question number 5 is to find out students' ability to solve contextual problems related to the algebraic expression of a pattern. The results obtained from the pretest and posttest cycle I are presented in Figure 6.

Based on Figure 6, it can be seen that in the pretest results, students have difficulty understanding patterns in the arrangement of objects and in the arrangement of numbers, have difficulty making generalizations of patterns in algebraic form, have difficulty explaining the meaning of variables from the context presented, and have difficulty solving contextual problems related to linear patterns and algebraic expression of a pattern. However, after applying learning with a didactical design that contains HLT linear patterns and algebraic forms of a pattern along with instructional tasks packaged in a worksheet, students

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experience an increase in understanding of these competencies. This is indicated by the reduced learning obstacles of students, as seen from the results of the cycle I post-test.

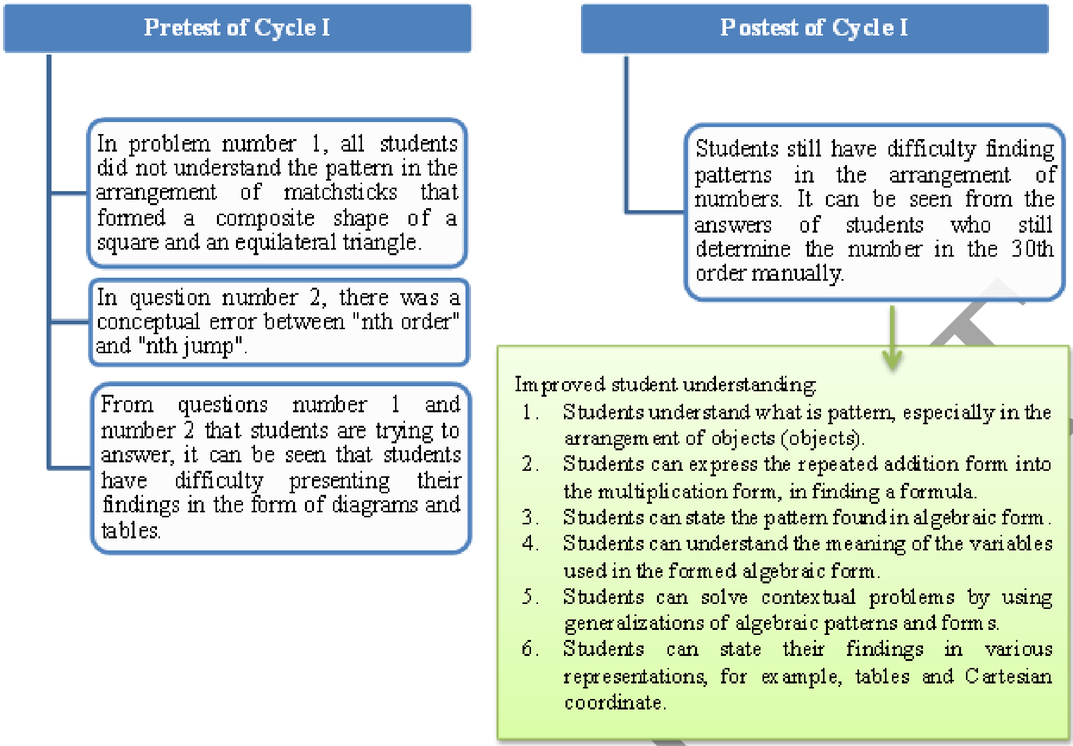

Figure 6. Diagram of Comparison of Pretest and Posttest Results in Cycle I

Based on Figure 7, the difficulties experienced by students during the pretest cycle I and cycle II are the same. However, the visible improvement from the results of the posttest cycles I and II are that students can apply the strategy of observing patterns in the number arrangement for question number 2. This indicates a reduction in student learning obstacles based on the HLT improvements made. All learning obstacles shown by students are types of epistemological obstacles. Epistemological obstacle is a student learning obstacle caused by students' limitations in applying the concepts learned (Brousseau, 1997,p 83). This can be seen from the difficulty of students applying the concept of multiplication that has been obtained in elementary school. Students also have difficulty applying and seeing the synchronization of the concept of patterns in the arrangement of objects with the concept of patterns in the number arrangement.

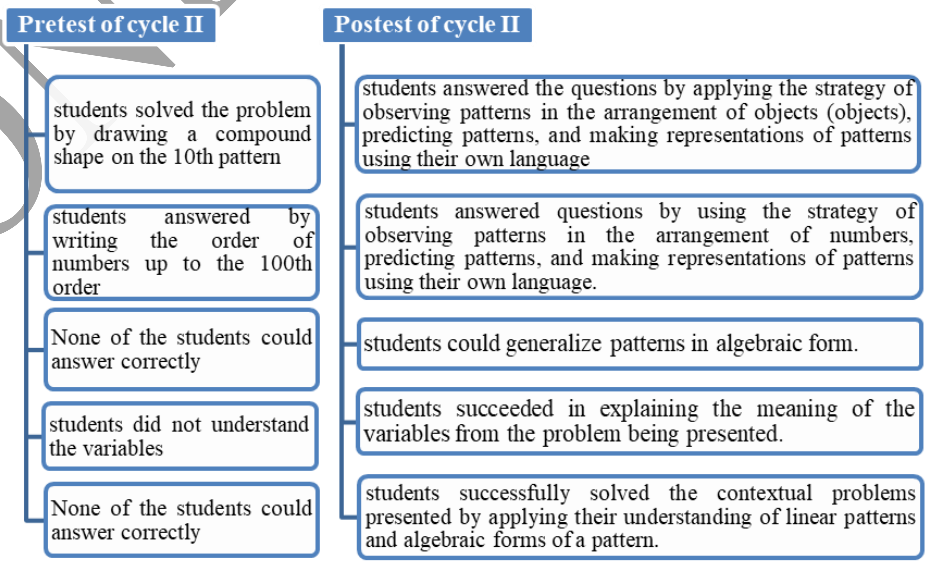

Figure 7. Comparison Diagram of Pretest and Posttest Results in Cycle II

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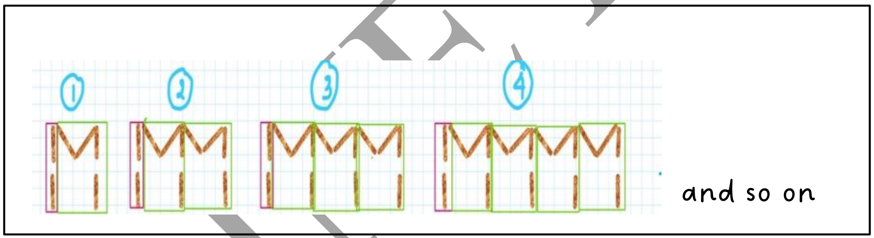
Based on the retrospective analysis that has been carried out in cycle I and cycle II, a didactic design of an algebraic expression of a linear pattern is produced. The resulting didactic design was designed based on student learning obstacles identified from the results of the pretest, pilot experiment, and posttest. The didactical design contains the pedagogical didactical actions given by the teacher, predictions of student responses, and anticipation of the predictions of student responses. Overall the application of the didactic design includes the process of abstraction, formulation, and validation carried out by students under guidance from the teacher (Suryadi, 2019). The learning trajectory and student worksheet provided are part of the didactic design. In its application, the abstraction process occurs in Activity I (work with matchsticks) and Activity II (work with number cards), the formulation and validation processes occur in Activity III (student production pattern), whereas in Activity IV (contextual problem) is the application the understanding gained by students regarding the concept of linear patterns and the algebraic expression of a pattern.

Activity I (Working with matchsticks)

Objective: To assist students in recognizing, predicting, and generalizing linear patterns in the arrangement of objects
Section 1 : Recognizing linear patterns in the arrangement of objects

Pedagogical didactic action:

1. The teacher divides students into small groups consisting of 5 students with heterogeneous abilities.
2. The teacher distributes 2 matchboxes and student worksheet for each group.
3. The teacher asks students to observe the worksheet instructions on pages 1 and 2 while the teacher explains important things in the activities to be carried out.
4. The teacher asks students to arrange the matchsticks into an arrangement of the letter M that is continuous and has a pattern.



Gambar 7. Pattern arrangement of letter M

Student response to action number 4:
There will be students who arrange the matchsticks into an arrangement of the letter M that is not connected (no parts of the letter M coincide with each other).

Anticipation of student's response to action number 4:
The teacher directs students to arrange the matchsticks according to the instructions on the student worksheet and emphasizes that there are parts of the letter M that coincide. If necessary, the teacher gives an example in compiling it.

5. The teacher asks students to observe how many matchsticks are needed to make each pattern and see how it differs from the previous pattern.

Student response to action number 5:

a. There are students who will only write down the number of matchsticks needed to make a pattern for the letter M in each pattern.
b. Students do not write down the results of observations of the number of matchsticks needed in repeated addition form.

Anticipation of student's response to action number 5:

a. The teacher asks students to look again at the description on the student worksheet and is assisted with direct guidance from the teacher, for example:

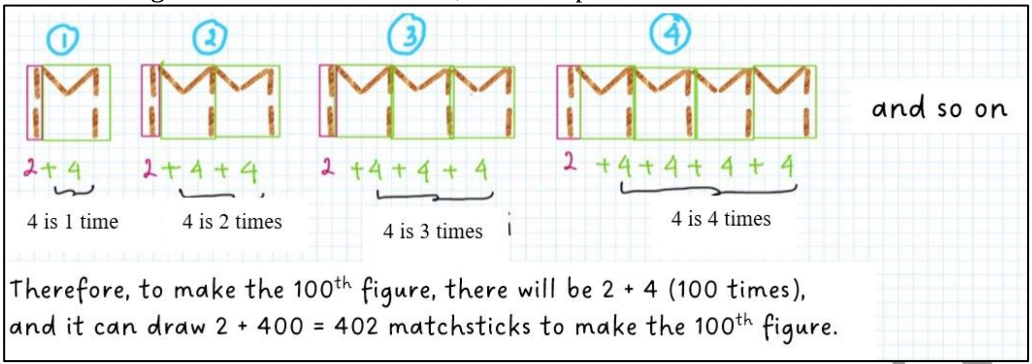


Figure 8. Emergence Model of the Letter Arrangement Pattern M

b. The teacher asks students to write down the addition of matchsticks from each pattern.

c. The teacher asks students to pay attention to the relationship from the previous pattern and write it in the form of repeated addition.

d. The teacher gives more intense directions to groups of students who are still having trouble writing pattern changes in the form of repeated addition.

6. The teacher directs students to convert the repeated addition form into the multiplication form.

Student response to action number 6:
There are students who are wrong in writing the multiplication form. For example, they have to write 4×6, but students write 6×4. This must be anticipated because the two writings have different meanings, and what is written must have a meaning that is in accordance with the circumstances of the pattern being observed.

Anticipation of the student's response to action number 6:
The teacher gives guidance to students in the form of a question "What number is added repeatedly?" and then “How many times is this number added up repeatedly?”. The teacher emphasizes the correct intonation on how to read sentences 4 × 6, namely the number 6 is added up 4 times, and what about the meaning in sentences 6 × 4.

7. The teacher asks students to observe changes in the generalization form of each written pattern, by directing students' focus on "states the things that are always changing" and "states the things that are always fixed".
8. The teacher directs students to think about generalizations for the 100th pattern by asking the question "How many certain numbers will be added up repeatedly in the 100th pattern?"
9. The teacher asks students to think of other forms of generalization in the same pattern by following the steps to find the generalization of the previous pattern.

Section 2 : Predicting and generalizing linear patterns on an array of objects

1. The teacher asks students to make arrangements of letters other than the letter M, the activity is carried out as in part 1. Each group chooses a different letter.
2. The teacher asks students to predict the pattern in the arrangement of letters made, by following the steps in the previous activity.
3. The teacher gives directions to students who experience problems in several parts of the activity.
4. The teacher directs students to predict patterns by writing down their observations about the number of matchsticks for each pattern, in the form of repeated addition.
5. The teacher directs students to write the repeated addition form into the multiplication form by paying attention to the multiplication concept that has been reminded in the previous activity.
6. The teacher asks students to generalize the 100th pattern from the arrangement of letters made.

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Activity II (Working with number of cards)

Objective: To assist students in recognizing, predicting, and generalizing linear patterns in number arrangements

Section 1 : Recognizing linear patterns in number sets

Pedagogical tactical actions:

1. The teacher conditions the class by dividing students into small groups consisting of 5 students with heterogeneous abilities (according to the group in the previous activity).
2. The teacher distributes 1 set of number cards and student worksheet for each group.
3. The teacher asks students to observe the worksheet instructions on pages 1 and 2 while the teacher explains the important things in the activities to be carried out.
4. The teacher asks students to arrange the number cards that have been distributed.
5. The teacher directs students to observe the change in numbers starting from card 1 to the numbers on the next cards, by asking the questions "How much must the first number be added to become the second number?", "how much must the second number be added to become the third number?", and so on.
6. The teacher directs students to predict the correct number on an empty number card.
   Student response to action number 6:
   Students immediately write down the number they are looking for without writing down how to get it.
   Anticipation of the student's response to action number 6:
   The teacher directs students to write down the results of their observations in the form of repeated addition.

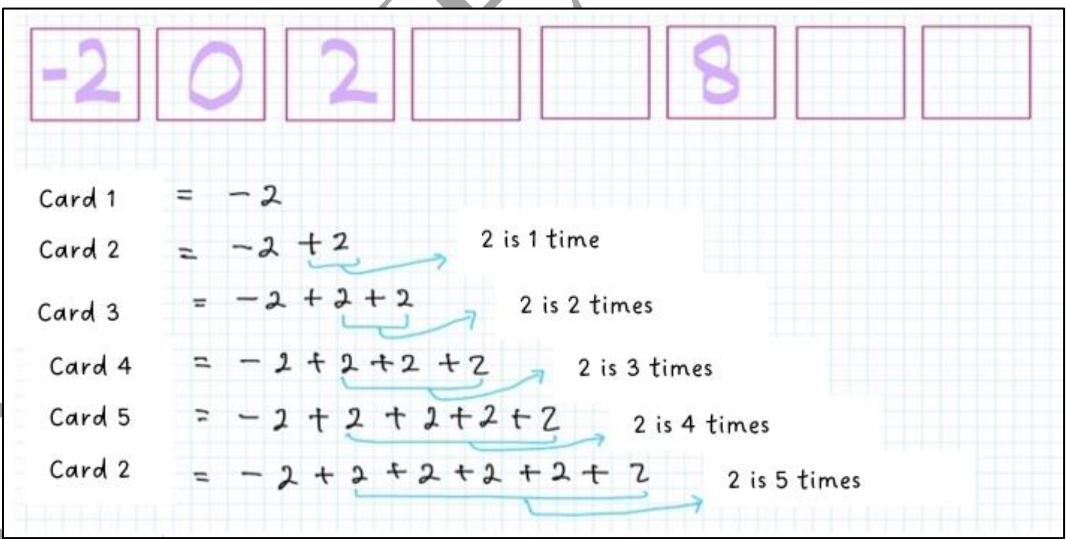

Figure 9. Number prediction on an empty even number card

7. The teacher directs students to convert repeated addition into multiplication form.
   Student response to action number 7:
   There are still some students who have difficulty and make mistakes in writing the multiplication form according to the correct meaning.
   Anticipation of student responses to action number 7:
   The teacher gives guidance to students in the form of a question "What number is added repeatedly?" and then "How many times is the number added repeatedly?"

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8. The teacher directs students to predict the correct number on the 50th card, if the 50th card is available, paying attention to changes in each pattern and the relationship between patterns.
   Student response to action number 8:
   Some students are constrained in understanding the relationship between patterns.
   Anticipation of student responses to action number 8:
   The teacher asks students to observe changes in the generalization form of each written pattern, by directing students' focus on "states that are always changing" and "states that are always fixed".

Section 2 : Predicting and generalizing linear patterns on number sets

1. The teacher distributes 1 set of blank number cards to each group.
2. The teacher asks students to arrange numbers that have a pattern on the number cards distributed, starting with any number.
3. The teacher directs each group to make a number arrangement that is different from the other groups.
4. The teacher directs students to predict the correct generalization for each pattern in the arrangement of numbers made, following the observation steps in the activity part 1.
5. The teacher asks students to make generalizations for the 50th card by paying attention to changes in each pattern and the relationship between patterns.

Activity III (Student production patterns)

Objective: Helping students to be able to write generalizations of linear patterns in algebraic expressions and understand the meaning of a variable as something whose value is not yet known.

Pedagogical tactical actions:

1. The teacher conditions the class by dividing students into small groups consisting of 5 students with heterogeneous abilities, according to the groups in the previous activity.
2. The teacher distributes 2 matchboxes and student worksheets for each group.
3. The teacher asks students to observe the worksheet instructions on pages 1 and 2 while the teacher explains important things in the activities to be carried out.
4. The teacher asks the students to arrange the matchsticks into an equilateral triangle that is connected to each other.
5. The teacher asks students to make various generalizations of different patterns from pattern 1, pattern 2, pattern 3, pattern 4, pattern 5, and pattern 100.
6. The teacher asks students to follow the steps in the previous activities in producing pattern generalizations.
7. The teacher asks students to observe numbers that are always changing from the generalization of each pattern and numbers that are always fixed.
8. The teacher asks students to make generalizations for the nth pattern by replacing the numbers that are always changing in the previous generalization by using the symbol of letter n.
   Student response to action number 8:
   Students feel confused with the use of the letter n.
   Anticipation of student responses to action number 8:
   a. The teacher explained that in the questions, there could be different questions. For example, determine the number of matchsticks in the 50th pattern, 100th pattern, 1000th pattern, and so on. The symbol for the letter n used can be replaced with the order of the pattern requested in the question. Example. In the question asking for the 100th pattern, all letters n in the resulting generalization must be replaced with the number 100, then the calculation is carried out. Then the resulting generalization is called the pattern generalization in algebraic form.
   b. In the student worksheet, it has been explained that the letter n in the resulting algebraic form is called a variable.
9. The teacher asks students to explain using their own language

Activity IV (Contextual problems)

Objective: Students can express a contextual situation in an algebraic expression.

1. The teacher conditions the class by dividing students into small groups consisting of 5 students with heterogeneous abilities, according to the groups in the previous activity.

2. The teacher distributes student worksheet for each group.
3. The teacher asks students to observe the worksheet instructions on pages 1 and 2 while the teacher explains important things in the activities to be carried out.
4. The teacher presents a contextual problem, namely "There are 6 chairs placed around a hexagonal table. Investigate how many chairs can be placed on the hexagonal tables are placed side by side (sides close together) (Gürbüz & Ozdemir, 2020).
5. The teacher asks students to observe how many chairs are needed if there is only 1 hexagonal table, how many chairs are needed if there are two hexagonal tables arranged side by side, and so on.
6. The teacher asks students to write down the results of student observations in the form of repeated addition as in the previous activities.
7. The teacher asks students to generalize patterns in the algebraic expression if there are n hexagonal tables.
8. The teacher asks the students to explain the meaning of the variable from the generalized pattern in the resulting algebraic expression.

Based on the results obtained from the research conducted, there are several identified learning obstacles for students in learning algebra in grade 8th, especially in the material of linear patterns and the algebraic form of a pattern. These learning obstacles can be seen from the answers written by students on the pretest and posttest answer sheets, as well as the results of students' work when carrying out a series of mathematical activities during the learning process. All learning obstacles shown by students are types of epistemological obstacles. Epistemological obstacle is a student learning obstacle caused by students' limitations in applying the concepts learned (Brousseau, 1997).

The resulting Learning Trajectory is the application of the Hypothetical Learning Trajectory (HLT) in cycle I. From the results of the implementation carried out in cycle I, there are several revisions made by researchers to the HLT to be reapplied in cycle II so as to produce the right Learning Trajectory in learning algebra class VII especially on the material of linear patterns and algebraic forms of a pattern.

Based on the learning process carried out, the results of the post-test, and the results of post-learning interviews with students, students experienced difficulties in understanding patterns in the arrangement of numbers. Therefore, the changes were made to activity 2 which is about working with number cards. The change made was by adding a guide to the student worksheet in writing the representation of the pattern observed on the number card. This is done because, based on the activities in cycle I, students have not been able to apply the concept of pattern representation to the arrangement of objects in making pattern representations of number arrangements. Based on interviews with students, these difficulties occurred because observing patterns in the arrangement of numbers was more abstract so these numbers could not be grasped directly. Consequently, it makes them difficult to determine what to do in finding patterns and making representations. With the additional guidance given in activity 2, students are more assisted in recognizing, predicting, and generalizing linear patterns in number arrangements, especially in making representations of the pattern. This improvement is effective because it can reduce learning obstacles experienced by students during learning and post-learning cycle II. Overall, from the application of HLT, there is no jump in student understanding. Students can follow the entire series of activities well, to be precise after the revision of the HLT implementation was held in cycle I. This indicates that the resulting learning trajectory is appropriate for conducting algebraic learning in grade 8th, especially on the topic of linear patterns and the algebraic expression of a pattern.

The sequence of learning trajectories in this study is different from the results of research conducted by Gürbüz & Ozdemir (2020) regarding learning trajectories in teaching the concept of variables, namely they contain four stages, namely i) students use literal expressions that are familiar to students in understanding algebraic forms, ii) students can find patterns and express them in the form of variables, iii) students can write in the form of algebraic expressions and replace these variables with numbers, and iv) students can find the concept of variables as something whose value is not yet known. From the resulting learning trajectory stages, students must first use literal expressions that represent an object so that students are familiar with the use of letter symbols in mathematical sentences, then enter pattern recognition. The resulting learning trajectory is local in terms of participants and the scope of the material. This is what causes the difference in the resulting learning trajectory sequence with the results of previous research.

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CONCLUSSION

The application of didactic design, which is carried out as an effort to reduce students' learning obstacles in learning algebraic expressions from linear patterns, can create didactic situations that provide opportunities for students to carry out thought processes as part of the learning process being carried out. However, with the formation of a didactic situation, there will be a variety of student responses that emerge as a manifestation of students' thinking about the pedagogical didactic actions given by the teacher. Applying the didactic design to the algebraic expressions of this linear pattern provides opportunities and space for students to find their algebraic forms involving a variable. In addition, students can find the meaning of the variables used. So that students no longer think that algebra is very abstract because students can find the concept with mathematical activities that involve students. The local learning trajectory produced in teaching algebraic forms from linear patterns can be passed by students well, and there is no jump in student understanding of all the activities designed. This also has a positive impact on students, namely reducing student learning obstacles in learning algebra in junior high school.

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