



Analyzing Elementary School Students' Misconceptions in Number Sense Using a Five-Tier Diagnostic Test

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ABSTRACT

Misconceptions related to the concepts underlying number sense represent one of the primary challenges faced by elementary school students in learning mathematics. This research analyzed the specific misconceptions encountered by elementary school students in Lubuklinggau, Indonesia, in relation to number sense, utilizing a five-tier diagnostic test. This qualitative descriptive study involved 28 Year 8 students from one of the Islamic School in Lubuklinggau who had previously studied number operations. Data collection was conducted using a five-tier diagnostic test and interviews. Data validity was ensured through triangulation. After administering the diagnostic test, the researchers cross-checked data through interviews and documentation. The data was analyzed in three stages: data reduction, data presentation, and data verification. The results indicate that (1) 27.6% of students experienced misconceptions; (2) the most prevalent type of misconception was the process-object error (31.5%); (3) the highest incidence of misconceptions occurred with the fifth indicator of number sense, assessing the reasonableness of a calculation (48.5%); and (4) the primary causal factor contributing to students' misconceptions was their own reasoning (0.97%).

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INTRODUCTION

Misconceptions are misunderstandings or errors stemming from incorrect conceptual understanding (Ojose, 2015). A misconception arises when an individual's understanding of a concept diverges from the accepted or standard interpretation, leading to misunderstandings that are not easily rectifiable (Nuraina et al., 2024). Such misconceptions often stem from students' prior learning experiences or ingrained incorrect beliefs (Ay, 2017). When misconceptions remain unaddressed, they

can negatively impact mathematics learning (Ojose, 2015), as one misconception can cascade into misunderstandings in subsequent mathematical concepts (Kurniati et al., 2018). This issue is especially relevant in mathematics education, where learning is often structured progressively, with each concept building on those introduced earlier (Hatip & Setiawan, 2021). Understanding and addressing students' misconceptions in mathematics is essential for teachers to support students' comprehension and reduce the likelihood of compounding errors.

One intriguing area of research on misconceptions in mathematics focuses on number sense. Number sense is fundamental in mathematics education and plays a critical role in everyday life (Purnomo et al., 2014; Yang & Lin, 2015; Fahlevi, 2022). Identifying and addressing misconceptions in students' number sense can significantly enhance their foundational mathematics skills. Number sense covers a student's ability to understand numbers, operations, and their relationships, as well as to apply these concepts flexibly in real-life contexts (McIntosh et al., 1992). It includes the capacity for flexible and intuitive thinking about numbers (Hadi, 2015) and enables mental calculations that do not rely on standard algorithms. With strong number sense, students can apply their understanding of numbers in solving mathematical problems using non-standard approaches (Fahlevi, 2022). A student with a well-developed sense of numbers is better equipped to utilize mathematics effectively in daily life (Singh et al., 2018). Thus, number sense extends beyond basic number recognition, counting, memorization, and procedural problem-solving. It consists of five key components: understanding the meaning of numbers, recognizing relative size, utilizing various representations, comprehending the effect of operations, and evaluating the reasonableness of calculations (Yang & Lin, 2015; Lin, 2016).

Despite its importance, misconceptions about number sense are prevalent, even among elementary school students. Numerous studies have explored these misconceptions (Yang & Ling, 2015; Lin et al., 2016; Yang, 2019; Uredi, 2022). Yang and Lin (2015) investigated number sense misconceptions using a four-tier diagnostic test with 10-11 years old students in Taiwan. Lin, Yang, and Li (2016) used a web-based two-tier diagnostic tool to examine student misconceptions, while Yang (2019) developed a three-tier diagnostic test for fifth-grade students. Uredi (2020) furthered this work by evaluating and developing curriculum-based strategies to address these errors. Common misconceptions include misunderstandings of decimal values, such as believing "longer is greater" (e.g., $4.03 > 4.3$) or "shorter is greater" (e.g., $0.2 > 0.25$) (Durkin & Rittle-Johnson, 2014). In the context of integers, some students mistakenly think a number's magnitude increases with its distance from zero (Kurniati et al., 2018). Similarly, in fraction comparisons, some students assume fractions like $\frac{5}{6}$ and $\frac{7}{8}$ are equivalent by focusing only on the difference between the numerator and denominator (Clarke & Roche, 2009).

Ben-Hur (2006) classified types of misconceptions into five categories: (1) Pre-Conception, the initial error that arises before a concept is fully understood; (2) Undergeneralization, a more specific form of pre-conception characterized by a limited understanding and restricted application of concepts; (3) Overgeneralization, where the application of concepts is too broad, making the applied rules irrelevant; (4) Modeling Error, observed when individuals imitate incorrect examples from prior mathematical representations; and (5) Prototyping Error, commonly seen in issues with understanding form conservation through standard examples.

Various factors can lead to these misconceptions. Students may develop misconceptions from their everyday experiences (Yangin et al., 2014). Suparno (2013) identified four primary sources of misconceptions: (1) students, (2) teachers, (3) textbooks and literature, and (4) teaching methods. Misconceptions originating from students may be due to prior knowledge, associative thinking, humanistic reasoning, incomplete reasoning, incorrect intuition, developmental stages, individual abilities, and levels of interest in learning (Suparno, 2013). Teachers who lack a thorough understanding of the material may unintentionally pass on misconceptions to students (Suprpto, 2020). Likewise, textbooks presenting incorrect information can create confusion (Lambi, 2009), and teaching methods focusing narrowly on a single concept may contribute to students' misunderstandings (Suparno, 2013).

Following a literature review on number sense misconceptions, this study to identify and analyze number sense misconceptions among elementary students in Lubuklinggau, Indonesia. There has been no prior research on misconceptions about this area among students in Lubuklinggau, Indonesia. This study employed a five-tier diagnostic test to analyze number sense misconceptions. The five-tier diagnostic is an extension of the four-tier diagnostic test. In the four-tier format, the first level records

the student's answer, the second level captures the student's confidence in that answer, the third level explores the student's rationale for the answer, and the fourth level assesses confidence in that reasoning (Yang & Lin, 2015). The five-tier diagnostic expands on this by adding questions on learning resources used to answer levels 1 and 3, enabling a deeper investigation into the sources of students' misconceptions (Febriyana et al., 2020).

METHOD

This study employed a descriptive qualitative research design to (1) identify elementary school students' misconceptions about number sense using a five-tier diagnostic test and (2) examine the causes of these misconceptions. The research was conducted at one of the Islamic primary school in Lubuklinggau, Indonesia during the first semester of the 2023/2024 academic year. The sample consisted of 28 Year 6 students selected through purposive sampling, ensuring alignment with the study's objectives. The selected students had previously been taught number operations.

The study began with preparing a five-tier diagnostic test, an instrument designed to assess students' misconceptions about number sense. This test comprises five response levels: the answer choice (T1), confidence in the answer choice (T2), the rationale for the answer (T3), confidence in the rationale (T4), and the learning source informing the answer (T5). The diagnostic test was initially piloted with students outside the study sample. Results from validity calculations showed a high validity, with an Aiken index of 0.8667. The reliability assessment yielded a Cronbach's Alpha coefficient of 0.704, indicating strong reliability. These results confirm that the five-tier diagnostic test is both valid and reliable.

Data collection involved administering the number sense misconception test to the sample and conducting follow-up interviews with students who demonstrated misconceptions. The interviews aimed to clarify and further understand the specific misconceptions held by these students. Out of the 28 students, 12 were interviewed. Student response documentation was also reviewed as part of the data collection. In qualitative research, ensuring data validity is essential to producing objective findings, and this study employed data triangulation to verify data accuracy (Hadi et al., 2021).

Data analysis followed the Miles and Huberman (1994) model, consisting data reduction, display, and conclusion drawing/verification. In the data reduction stage, student responses were categorized based on each tier of the diagnostic test. To identify student misconceptions, the CRI (Certainty of Response Index) scale was applied at the second and fourth levels (T2 and T4) for confidence in answers and reasoning. The data display stage presented the percentage of student responses and the percentage of misconceptions identified in each category from the previous stage. This allowed for assessing the number of students with misconceptions in number sense. Data were also presented as a descriptive narrative, supplemented by theoretical analysis of misconceptions and their causes. In the final stage, verification, data were examined to conclude the types of misconceptions experienced by students in number sense and their underlying causes.

The CRI scale used in this study ranges from 1 to 4 for each question. This scale, combined with criteria for conceptual understanding, misconception, and lack of understanding, helps to clarify students' comprehension levels. A score of 1 indicates limited understanding or lack of knowledge, while a score of 4 reflects a strong understanding and confidence in responses. Detailed criteria are presented in Table 1.

Table 1. Criteria for students with conceptual understanding, misconceptions, and no conceptual understanding

| Criteria | Low CRI (Scale 1-2) | High CRI (Scale 3-4) |
|----------------|--|---|
| Correct Answer | Correct answer but Low CRI means the students do not understand the concept (Guessing) | Correct answer and high CRI shows that the student has mastered the concept |

| | | |
|------------------|--|---|
| Incorrect Answer | Incorrect answer and low CRI shows that the student does not understand the concept. | Incorrect answer but high CRI high means the student has a misconception. |
|------------------|--|---|

Table 1 demonstrates that a scale of 1-2 indicates students are guessing their answers with little regard for accuracy, while a scale of 3-4 reflects high confidence in their responses. When students score high on the CRI scale (3-4) and answer correctly, this suggests they understand the concept. However, if their answer is incorrect, it indicates a misconception (Tayubi, 2005). Therefore, students are categorized as experiencing misconceptions when they provide incorrect answers with a high level of confidence.

RESULTS AND DISCUSSION

Data analysis was conducted based on the results of the five-tier diagnostic test, evaluating student misconceptions about number sense. Table 1 presents the summary of the results based on indicators (in percent) in categories that refer to the Certainty of Response Index (CRI).

Table 2. Summary of Results Based on CRI Category (%)

| No. | Indicator | No soal | PK | | ME | MK | TPK |
|--------------------|--|---------|------|------|------|------|-----|
| | | | P | S | | | |
| 1 | Understanding the meaning of numbers and their operations | 3 | 18 | 28 | 11 | 36 | 7 |
| | | 4 | 29 | 29 | 11 | 29 | 4 |
| 2 | Recognizing the relative size of a number | 1 | 54 | 21 | 0 | 18 | 7 |
| | | 2 | 32 | 54 | 7 | 0 | 7 |
| 3 | Being able to use different representations | 5 | 25 | 36 | 25 | 14 | 0 |
| | | 6 | 14 | 32 | 14 | 25 | 14 |
| 4 | Recognizing the relative effects of operations on numbers | 7 | 7 | 29 | 21 | 32 | 11 |
| | | 8 | 43 | 21 | 7 | 25 | 4 |
| 5 | Being able to assess the reasonableness of calculation results | 9 | 11 | 29 | 11 | 29 | 14 |
| | | 10 | 11 | 7 | 4 | 68 | 11 |
| Average percentage | | | 24.4 | 28.6 | 11.1 | 27.6 | 8.6 |

Notes: PKP = Fully understanding the concept
 PKS = Partially understanding the concept
 ME = Guessing
 MK = Misconception
 TPK = Not understanding the concept

Table 2 shows that students with partial concept understanding (PKS) represent the largest proportion (28.6%), followed by those with misconceptions (27.6%), students who fully understand the concept (24.4%), students who make guesses (11.1%), and students who do not understanding the concept (8.6%). Although not the largest, the percentage of students experiencing misconceptions is notably high compared to other categories. This finding aligns with other studies indicating that misconceptions are common among students solving number sense problems (Widyasari et al., 2021; Hazril et al., 2022). The researcher further categorized student misconceptions by specific number sense indicators to identify which indicators had the highest misconception rates, as shown in Table 3.

Table 3. Misconception results based on number sense indicators

| No. | Indicator | Misconception (%) |
|-----|--|-------------------|
| 1 | Understanding the meaning of numbers and their operations | 32.5 |
| 2 | Recognizing the relative size of a number | 9 |
| 3 | Being able to use different representations | 19.5 |
| 4 | Recognizing the relative effects of operations on numbers | 28.5 |
| 5 | Being able to assess the reasonableness of calculation results | 48.5 |

Table 3 illustrates that the highest incidence of misconceptions occurs in the fifth indicator—assessing the reasonableness of a calculation (48.5%). Nearly half of the students in the class demonstrated misconceptions in this area. The indicator relating to understanding the meaning of numbers and their operations ranks second, despite being a fundamental aspect of number sense. Meanwhile, the indicator for recognizing the relative size of numbers shows the smallest percentage of misconceptions (9%).

Among the 27.6% of students who exhibited misconceptions, 48.5% did so in assessing the reasonableness of calculation results. This finding indicates that many students struggle to mentally estimate and evaluate whether a calculation’s result is plausible. An example of a student’s response demonstrating this misconception is shown in Figure 1.

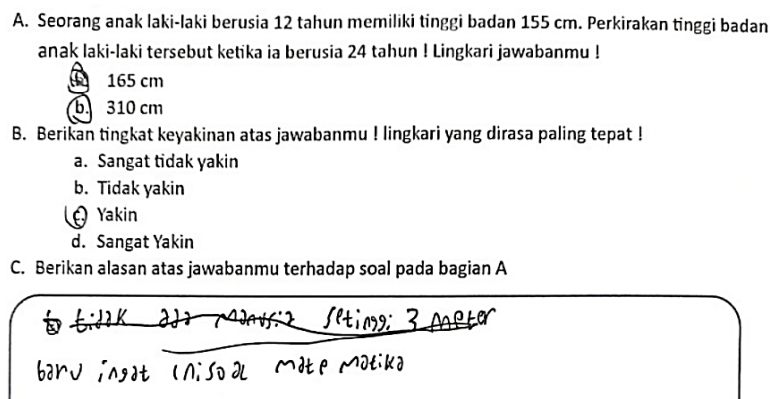


Figure 1. Student's response on the reasonableness indikator

In Figure 1, the student initially doubts that a human could be 3 meters tall. However, upon realizing this is a math problem, they proceed with a standard calculation approach, estimating height through proportional reasoning. This process leads the student to an incorrect conclusion, highlighting a misconception. Further analysis was conducted to categorize the types of misconceptions among students, with the results presented in Table 4.

Table 4. Percentage of Students with Misconceptions by Type

| Misconception type | Percentage |
|----------------------|------------|
| Pre-Conception | 13 |
| Undergeneralization | 6.5 |
| Overgeneralization | 18.4 |
| Modelling Error | 20.1 |
| Process-Object Error | 31.5 |
| Prototyping Error | 10.53 |

Table 4 reveals that the most common type of misconception among students regarding number sense is the process-object error (31.5%) of cases, followed by modeling errors (20.1%), overgeneralization (18.4%), preconception errors (13%), prototyping errors (10.53%), and, undergeneralization (6.5%). An example of a student's response demonstrating the process-object error is shown in Figure 2.

- A. Tanpa mengalikan kedua bilangan, susunlah hasil operasi berikut dari yang terkecil hingga terbesar
- $P = 16 \times \frac{3}{3}$; $Q = 16 \times \frac{1}{2}$; $R = 16 \times \frac{3}{2}$; $S = 16 \times \frac{2}{3}$
- a. P Q R S
 b. Q S P R
- B. Berikan tingkat keyakinan atas jawabanmu ! lingkari yang dirasa paling tepat !
- a. Sangat tidak yakin
 b. Tidak yakin
 c. Yakin
 d. Sangat Yakin
- C. Berikan alasan atas jawabanmu terhadap soal pada bagian A

karena p hasilnya adalah 16 dari perhitungan saya jadi
 & Saya menjawab A.

Figure 2: Student's response with process-object error misconception

As shown in Figure 2, the student provided an incorrect answer, believing it to be correct, and explained that "P is 16 from the calculation, so the answer is A." The researcher further explored the student's reasoning in the following interview.

- P* : Why did you choose A?
S : Because I calculated that P is 16, so the next one must be greater than 16. The first P is 16.
P : Why do you think anything other than P is greater than 16?
S : Because P is multiplied by $\frac{3}{3} = 1$, and the other one is not, so it must be greater.

This misconception, classified as a process-object error, reflects the student's misunderstanding of calculating with whole numbers and fractions (Ben-Hur, 2006). Modeling errors are the second most frequent type of misconception. Figure 3 illustrates this error type from one of the students' answers.

- A. Dengan memperkirakan, lingkari jawaban dari operasi berikut :
- $350,4 \times 0,495$
- a. 173,448
 b. 17,3448
- B. Berikan tingkat keyakinan atas jawabanmu ! lingkari yang dirasa paling tepat !
- a. Sangat tidak yakin
 b. Tidak yakin
 c. Yakin
 d. Sangat Yakin
- C. Berikan alasan atas jawabanmu terhadap soal pada bagian A

karena angka dibelakang koma (.) berjumlah 4 dan yg
 b angka dibelakang koma (.) berjumlah 4

Figure 3. Student Responses with Modeling Error Misconceptions

As shown in Figure 3, the student selected and defended their answer based on counting the digits after the decimal point, which totaled four. The researcher further investigated this reasoning through the following interview.

- P* : Are you confident about your reasoning for the answer to problem nine?
S : Yes, that's what I understand ... if there's a number multiplied, you just count how many digits are behind the decimal to determine where the decimal point should go ...
P : Did it occur to you that 0.495 is close to 0.5 or half, so the result might be estimated as half of 350?
S : I didn't know that...

This response reflects a misconception classified as a modeling error, in which the student replicates an incorrect method from a prior mathematical representation without understanding the reasoning behind it (Ben-Hur, 2006). This type of error suggests that many students struggle with modeling in mathematics, often viewing mathematical procedures as unchanging doctrines provided by the teacher, who they believe cannot be mistaken (Kusmaryono et al., 2020).

In cases of overgeneralization errors, the researcher found instances where students assumed that any multiplied number is always larger than an added number. This misconception is illustrated in Figure 4.

A. Tanpa melakukan perhitungan, manakah operasi yang menunjukkan hasil yang lebih besar ?
Lingkari jawabanmu !
 a. 86×4
 b. $86 + 87 + 88 + 89$

B. Berikan tingkat keyakinan atas jawabanmu ! lingkari jawaban yang dirasa paling tepat !
a. Sangat tidak yakin
b. Tidak yakin
c. Yakin
 d. Sangat Yakin

C. Berikan alasan atas jawabanmu terhadap soal pada bagian A

karena hasil di kali itu lebih besar

Figure 4: Student Responses with Overgeneralization Misconceptions

Based on Figure 4, students seem to believe that the result of any multiplication is always greater than that of addition. This misunderstanding leads them to incorrectly select option A, while firmly believing it to be correct. The researcher confirmed this reasoning through an interview with the student, as shown in the following transcript.

- P* : Farhan, for the first problem, you choose option A, which involves multiplication. You seem confident about this answer. Can you explain why?
S : Because when you multiply, the result is always greater than when you add, so the answer is A.

The student's response demonstrates an overgeneralization error. The student incorrectly applies the concept of multiplication without understanding its contextual relevance (Ben-Hur, 2006). Additionally, in cases of pre-conception misconceptions, some students believe that no number exists between two decimal numbers, as illustrated in Figure 5. However, not all students exhibiting this type of misconception make the same error across different problems.

- A. Berapa banyak bilangan desimal yang berbeda diantara 0,25 dan 0,26 ? Lingkari jawabanmu !
- Tidak ada
 Banyak
- B. Berikan tingkat keyakinan atas jawabanmu ! lingkari yang dirasa paling tepat !
- Sangat tidak yakin
 Tidak yakin
 Yakin
 Sangat Yakin
- C. Berikan alasan atas jawabanmu terhadap soal pada bagian A

karena tidak banyak biangan desimal yg berbeda.

Figure 5: Students with Pre-Conception Type Misconceptions

The student responses in Figure 5 indicate incorrect answers paired with high confidence, as students believe there are limited distinct decimal numbers. The researcher confirmed this reasoning through interviews, as shown in the following transcript.

P : What do you mean by “not many different decimal numbers” in your answer?


S : The number of digits after the decimal point is just two.

P : Can you explain what you mean by two?

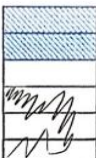
S : It's 0.2, so it's the same as 0.2 ... then it's sequential, after 5 comes 6 ... so there aren't any other numbers.

The researcher classified this student's misconception as a pre-conception error. A pre-conception misconception reflects an initial misunderstanding before a correct grasp of the concept is formed. This type of error often occurs in fundamental aspects of concept understanding (Ben-Hur, 2006). Misconceptions about decimal numbers may arise because students lack an understanding of decimal placement on the number line (Heldi & Nurjanah, 2019). Additionally, students might mistakenly interpret the value “2” after the decimal as an “amount.” The researcher also noted another interesting student response to a similar question, as shown in Figure 6.

- A. Manakah dari gambar dibawah ini yang menunjukkan nilai pecahan $\frac{2}{3}$!
- a.



b.


- B. Berikan tingkat keyakinan atas jawabanmu ! lingkari yang dirasa paling tepat !
- Sangat tidak yakin
 Tidak yakin
 Yakin
 Sangat Yakin
- C. Berikan alasan atas jawabanmu terhadap soal pada bagian A

Yakin

Figure 6. Student Responses Exhibiting Misconceptions

In Figure 6, students seem to have misconceptions, believing there are no suitable option and therefore creating their own response by crossing out three parts of the figure to represent $\frac{2}{3}$. It is

suspected that the students do not understand that option A is equivalent to $\frac{2}{3}$. This was further investigated through interviews with the students. The following is an excerpt from the interview.

- P : Why did you cross out option B?
S : So, there would be an answer ... because nothing in the picture matched $\frac{2}{3}$.
P : What fraction does option A represent?
S : $\frac{4}{6}$
P : is $\frac{2}{3}$ and $\frac{4}{6}$ different?
S : Yes, it's different

This misunderstanding is classified as a prototyping error, where students assume that a standard example of a concept is the only valid representation of that concept (Ben-Hur, 2006). Finally, in the case of undergeneralization misconceptions, some student responses indicated a limited understanding of fractional numbers, as demonstrated in Figure 7.

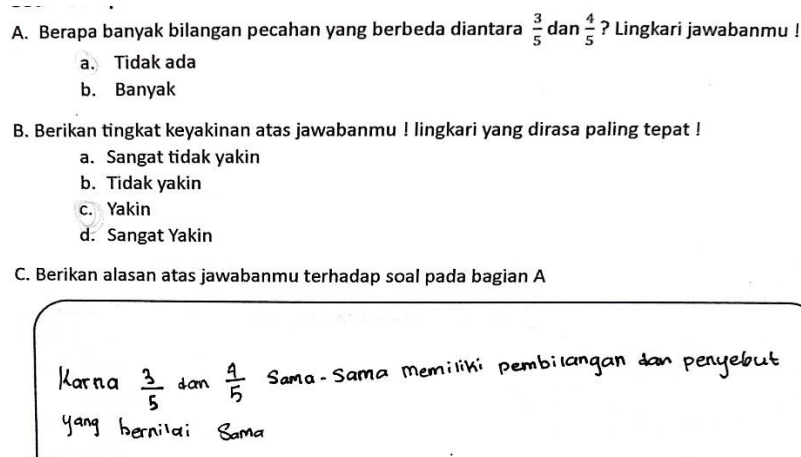


Figure 7. Students with Misconceptions in Problem Four

Figure 7 shows that the student confidently provides an incorrect answer, explaining it as valid because both fractions ($\frac{3}{5}$ and $\frac{4}{5}$) have numerators and denominators of the same value. The researcher confirmed these results with the student through the following interview.

- P : What do you mean by having the same numerator and denominator? Could you explain?
S : It's the same, 5...
P : The denominator is indeed the same, but what about the numerator?
S : Oh yes, I see, only the denominators are the same; the numerators are different, but they're consecutive, so there are no other numbers in between.

The student's response illustrates an undergeneralization misconception, a more specific form of pre-conception error, reflecting a limited understanding and restricted ability to apply concepts (Ben-Hur, 2006).

The researcher also explored the sources of these misconceptions based on students' answers to a fifth-level question (T5): "Where did you get the information used, and what is your reasoning for this answer?" Analysis revealed that students' own thinking was the primary cause of misconceptions. Of the 76 misconceptions identified, 74 stemmed from students' own reasoning, with only one from a teacher and one from peers. This means that 97.36% of the misconceptions originated from students' internal thought processes, while teacher and peer influences each accounted for only 0.013%. Further investigation suggested that students' misconceptions might have initially been influenced by a teacher's

misunderstanding of number sense concepts. Given that students' thought processes are shaped over years of cumulative learning (Ojose, 2015), it is crucial to introduce number sense understanding early to minimize and prevent entrenched misconceptions.

CONCLUSION

Based on the findings, it is concluded that, among the 28 students sampled, each of whom answered 10 questions using the five-tier diagnostic test, 27.6% demonstrated misconceptions. The most common misconception was related to the fifth indicator (48.5%), which assesses the reasonableness of a calculation. This finding suggests that nearly half of the students struggled with evaluating the reasonableness of calculations. The most frequent type of misconception observed was the process-object error (31.5%), with the primary source of misconceptions being students' own thinking, accounting for 97.36%. These findings indicate the importance of teachers addressing misconceptions through classroom instruction that fosters and reinforces number sense skills.

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