Available online at: http://journal.uny.ac.id/index.php/pythagoras
PYTHAGORAS: Jurnal Matematika dan Pendidikan Matematika, 18 (2), 2023, 176-186

Hypothetical Learning Trajectory of Sector Area and Arc Length Using the Clockwork Context

Ellis Salsabila*, **Mimi Nur Hajizah**
Study Program of Mathematics Education, Faculty of Mathematics and Natural Sciences, Universitas Negeri Jakarta, East Jakarta, Indonesia
* Corresponding Author. E-mail: ellis@unj.ac.id

ARTICLE INFO

Article History:
Received: 29-Oct. 2023
Revised: 19-Feb. 2024
Accepted: 21-Mar. 2024

Keywords:
Arc length, clockwork, hypothetical learning trajectory, realistic mathematics education, sector area

ABSTRACT

Students' understanding on the concept of a circle especially the relationship among central angles, arc lengths, and sector areas of the circle based on learning obstacle and learning trajectory need to be developed well by creating a good didactic design. The research aims to develop a series of learning activities to explore the concept of arc length and sector area of the circle as a proportion problem in the form of a hypothetical learning trajectory. The research method of the present study used is a qualitative research method in the form of Didactical Design Research which goes through three stages, namely: (1) the preliminary design; (2) teaching experiment; and (3) retrospective analysis. This study focused on the first stage. The HLT was designed using a realistic mathematics education (RME) approach. There are three anticipated learning obstacles in the design of the activities, including errors related to the concept of the relationship among arc lengths, circle circumferences, sector areas, and circle areas; errors related to the concept of the relationship among central angles, arc lengths, and sector areas of a circle; as well as errors related to the connection of concepts in circle material with other mathematical material; and students' difficulties in problem-solving questions. This study has the implication that the hypothetical learning trajectory (HLT) of sector areas and arc lengths can be used as an alternative design and development learning that can be used to minimize students' learning obstacles on the concepts of sector areas and arc lengths of the circle and can develop students' mathematical abilities.

Pemahaman siswa terhadap materi lingkaran khususnya hubungan antara sudut pusat, panjang busur, dan luas juring lingkaran berdasarkan hambatan pembelajaran dan lintasan belajar perlu dikembangkan dengan baik dengan menciptakan desain didaktik. Penelitian bertujuan untuk mengembangkan rangkaian kegiatan pembelajaran untuk mendalami konsep luas sektor dan panjang busur sebagai permasalahan perbandingan dalam bentuk hipotetis lintasan belajar. Metode penelitian yang digunakan adalah metode penelitian kualitatif berupa Penelitian Desain Didaktis yang melalui melalui tiga tahap, yaitu: (1) preliminary design; (2) eksperimen pengajaran; dan (3) analisis retrospektif. Penelitian ini berfokus pada tahap pertama. Hipotetis lintasan pembelajaran dirancang dengan menggunakan pendekatan pendidikan matematika realistik. Terdapat tiga hambatan pembelajaran yang diantisipasi dalam perancangan kegiatan, antara lain kesalahan terkait konsep hubungan panjang busur, keliling lingkaran, luas bidang, dan luas lingkaran; kesalahan terkait konsep hubungan sudut pusat, panjang busur, dan luas busur lingkaran; serta kesalahan terkait keterhubungan konsep pada materi lingkaran dengan materi matematika lainnya; dan kesulitan siswa dalam penyelesaian masalah. Penelitian ini memiliki implikasi bahwa Hypothetical Learning Trajectory (HLT) yang dikembangkan dapat dijadikan salah satu alternatif rancangan dan pengembangan pembelajaran yang dapat digunakan untuk meminimalisasi learning obstacle siswa pada konsep panjang busur dan luas juring lingkaran serta dapat mengembangkan kemampuan matematis siswa.

How to Cite:
Salsabila, E., & Hajizah, M. N. (2023). Hypothetical learning trajectory of sector area and arc length using the clockwork context. *Pythagoras: Jurnal Matematika dan Pendidikan Matematika, 18*(2), 176-186. https://doi.org/10.21831/pythagoras.v18i2.67027

https://doi.org/10.21831/pythagoras.v18i2.67027

INTRODUCTION

Mathematics is a science that is built from a variety of structured topics, consisting of several topics that are interrelated with one another such as geometry, algebra, statistics, and trigonometry (National Council of Teachers of Mathematics, 2020). Geometry is an important branch of mathematics for students to master since the application of geometry can be put into everyday life. Geometry, another important branch of Mathematics, has a place in education for the development of critical thinking and problem solving; furthermore, geometrical shapes that are mostly discussed in Geometry are also included in parts of everyday life as they appear almost everywhere (Serin, 2018).

One of the concepts included in the geometric domain is the circle. Diana et al. (2020) pointed out that the characteristics of students in learning obstacles on the circle material is divided into several categories, including ontogenical, epistemological, and didactical. An ontogenical obstacle occurs because there is a leap in the students' creative thinking processes in problem solving; an epistemological obstacle occurs because of the limited contexts known to students; while a didactical obstacle occurs in some basic concepts taught by the teacher and has a major impact in the formation of students' concepts (Diana et al., 2020). In a learning trajectory, how students learn to solve problems related to circles contributes to the cognitive psychology theories (Mayer, 2001). Hence, students' creative thinking abilities on the concept of a circle based on learning obstacle and learning trajectory need to be developed well. It is also necessary to know in advance the prerequisite material required before solving the problems about the circle in order to obtain a good didactic design so that it can be implemented effectively (Hasibuan et al., 2019).

To help students in understanding the concepts of sector areas and arc lengths, HLT needs to be designed. HLT consists of learning goals, learning activities, and hypothetical learning process (Simon, 1995; Baroody, 2022). In order to support students' development in understanding the concepts of arc lengths, circle circumferences, sector areas, and circle areas, the realistic mathematics education (RME) approach was applied. The RME approved was chosen because of the underlying theory for this study relates to its potential to address issues of instructional designs (Plomp, 2013). With the RME approach, students will be able to learn mathematical concepts using real problems that are close to their daily lives (Suwarsono, 2001). By doing mathematical activities, students are stimulated to solve real problems using their informal knowledge (horizontal mathematization). Further in the study, the students will gradually be facilitated to rediscover formal mathematical concepts through vertical mathematization (Duyen & Loc, 2022). The novelty of this present study lies in the use of the clockwork context in bridging the concept of proportion with that of arc lengths and sector areas of a circle.

METHOD

A didactical study design is used in this study with the aim to provide the researchers with useful students' perspectives for developing learning theories. The outcomes are expected to be specifically valuable. The didactical design is developed through three stages, namely: (1) experiment preparation; (2) teaching experiment; and (3) retrospective analysis (Van Eerde, 2013). In the experiment preparation phase, or also known as the preliminary phase, the importance of designing the possible learning processes that can occur during instructional activities in the classroom are highlighted, and formulated in the form of an initial design. In the teaching experiment phase, the initial design is implemented. The learning activities allow the researchers to investigate whether students' mental activity is as anticipated. Meanwhile, in the retrospective analysis phase, an improved local instruction theory based on what was learned from a series of micro-design cycles in a teaching experiment will be reconstructed.

From the three stages of the didactical design development, this current study only focuses on the first stage, which is the preliminary design, since the main purpose of this article is in the development of a series of learning activities to delve the concept of sector areas and arc lengths as proportional problems in the form of HLT. In designing the student's HLT for the learning sectors area and arc length, the clockwork context will be used with a realistic mathematics education (RME) approach. The RME is a mathematical approach that emphasizes on the use of student's knowledge of mathematical concepts in actual real life situations (Van den Heuvel-Panhuizen et al., 2020; Anggraini et al., 2020). The application of the RME approach involves the understanding of contextual problems, explaining problems, solving problems, comparing and discussing answers, and concluding (Lady et al., 2018).

The preliminary design phase begins with identifying the learning obstacle, clarifying the objectives of learning mathematics combined with anticipating teaching experiments by imagining how the teaching and learning process can be realized in the classroom. An explicit formulation of a hypothetical learning trajectory which consists of three components will be produced. Those components are: (a) learning objectives for students; (b) instructional activities that are planned and the learning media that will be used; and (c) the alleged learning process that anticipates how students' thinking and understanding can develop when instructional activities are applied in the classroom. This hypothetical learning trajectory is open to adaptation based on inputs from students and evaluation of students' real understanding. This HLT also reflects the importance of anticipating possible learning processes that can occur when instructional activities are applied in the classroom. To identify the students' learning obstacles, several problems about the circle had been given to 34 students who had previously received learning material about circles. Students were selected purposively based on the criteria: having an important role in mathematics learning at school, having valuable knowledge in accordance with research studies, and having the desire to work together and share information about research studies. The questions given are problem solving questions that can reveal students' learning obstacles related to the concepts of the circle as well as arc lengths and sector areas of a circle.

RESULTS

The HLT design about sector areas and arc lengths begins with strengthening students' understanding of related materials that students have previously learned, including the elements of a circle, the concepts of circumference and area of a circle, as well as the concept of proportion. The circle elements that are closely related to this discussion are the central angles of the circle and circular arcs.

Students must understand that the arc of a circle is part of the circle and the arc is part of the area bounded by the circle so that the length of the arc is part of the circumference of the circle and the area of the arc is part of the area of the circle. The introduction of the concept of the relationship between the central angle, the length of the arc, and the area of the cross section of the circle, in this didactical design, begins with strengthening the concept of proportion that students have acquired in previous learning experiences. Furthermore, students are guided to be able to draw conclusions about the relationship between the central angle, the arc length, and the area of the circular section without having to stick to the formulas that have been given in the textbooks they have. There are three activities of lesson designing.

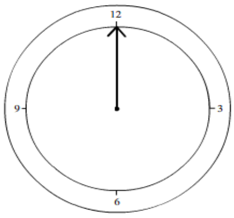
Lesson Design of Activity 1

Anticipated learning obstacles in the first activity include errors related to the concept of the relationship between arc lengths and circle circumferences, as well as sector areas and circle areas. There are two learning objectives contained in this design, namely: (a) students are able to determine the ratio between the size of the central angle and the size of the angle of one rotation and the ratio of the length of the arc to the circumference of the circle, and conclude the relationship between the two, and (b) students are able to determine the ratio between the magnitude central angle with the size of the angle of one revolution and the ratio of the area of the sector to the area of the circle, as well as summarizing the relationship between the two. The questions contained in the design direct students to understand that the arc of a circle is part of a circle and the arc is part of the area bounded by a circle so that the length of the arc is part of the circumference of the circle and the area of the arc is part of the area of the circle. In the current study, five example situations are presented for Lesson Design of Activity 1.

Situation 1 is designed by presenting a contextual problem that encourages a mental action to help students in recalling their memories about the elements of circles used in contexts related to circle material, namely the central angle of a circle (Scheme 1). This value is then used to determine the ratio of the central angle to the angle of one complete rotation. The central angles in question are the quarter circle angle ($90°$) and the semicircle angle ($180°$). These two angles are angles that are relatively familiar to students and are relatively easy to compare with the angles of one complete rotation.

Question 1
Please take a look at the circle-shaped clock below. If the minute hand moves for 15 and 30 minutes, what is the current position of the minute hand? Create a sketch by completing the picture.

Pay attention to the angle formed by the minute hand before and after moving, the corner point is located at the center of the hour. Does the angle represent an element of a circle? What is the circle element? Determine the size of the angle.



Scheme 1. An example of question for Situation 1 of lesson design of activity 1

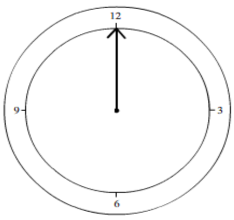
Following Situation 1, the next step is presented, which is Situation 2. This situation leads students to determine the ratio of the central angle to the angle of one complete rotation (Scheme 2). This comparison value will then be connected to the comparison value of the length of the arc to the circumference of the circle obtained from the next situation. Teachers should provide scaffolding to direct students to remember the concept of comparison.

Question 2
Determine the comparison of the results you obtained in Question 1 with an angle of one full turn.

Scheme 2. An example of question for Situation 2 of lesson design of activity 1

Next is Situation 3 where students are asked to name the elements of the circle that represents the given situation. Students are also asked to determine the length of the path which is none other than the circumference of the circle (Scheme 3).

Question 3
Please take a look at the circle-shaped clock below. If the minute hand moves one full rotation until it returns to its original position, pay attention to the path it takes. Does the path represent a circular element? What is the circle element? Determine the length of the path traveled by the minute hand.



Question 4
Please take a look at the circle-shaped clock in Question 3. If the minute hand moves 15 minutes and 30 minutes, pay attention to the path it takes. Does the path represent a circular element? What is the circle element?

Scheme 3. Two examples of questions for Situation 3 of lesson design of activity 1

In the situation that is represented by Question 3, students are not given information regarding the measurements needed, such as the length of the needle (radius of the circle). Students are given the freedom to use various methods to determine the length of the path traversed by the needle (circumference of the circle). Students can directly measure the circumference with a piece of thread, measure the length of the needle with a ruler, or calculate the length of the fingers with a value. Meanwhile, the situation in Question 4 is also related to the situation in Question 3. This situation is given to raise students' awareness that a circular arc is part of a circle; furthermore, that the length of a circular arc is part of the circle's circumference.

In Situation 4, students are directed to conclude the activities that have been carried out in Situations one to three. Conclusions can be obtained by discussing with friends. Apart from that, teachers also need to provide scaffolding so that the intended conclusions can be understood by students; so that students understand that the ratio of the central angle to the angle of one rotation is the same as the ratio of the length of the arc to the circumference of the circle (Scheme 4).

Question 5
Pay attention to the results you got in Questions No.1 to No.4. What can you conclude about the relationship between a central angle and an arc length?

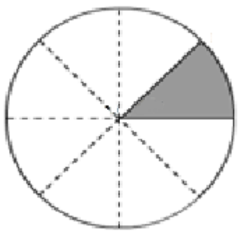
Scheme 4. An example of question for Situation 4 of lesson design of activity 1

The last situation in Lesson Design of Activity 1 is Situation 5 where the questions given represent a contextual problem that encourages a mental action to recall students' memories about the elements of a circle and the concept of comparison used in contexts related to circle materials, namely the central angle of a circle. This situation is similar to Situation 1, but with more complex angle measurement calculations.

Scheme 5 presented below is an example of how Situation 5 is developed into a problem-solving question. In this situation, students begin to be directed to the concept of the area of a circle as part of the area of a circle while still using the concept of comparison.

Question 6
Please take a look at the circle-shaped cake that is cut into 8 pieces below. If it is known that the radius of the cake is 7 cm, determine the total surface area of the cake and the surface area of each piece of cake.

Determine the ratio of the surface area of each piece to the surface area of the whole cake. Does the piece of cake represent a circular element? What is the circle element?



Scheme 5. An example of question for Situation 5 of lesson design of activity 1

From these problems given, it is predicted that several student responses will arise. Regarding the situation of determining the total surface area of the cake, the predicted response that emerged was that students used the formula for the area of a circle that they had obtained in previous learning experiences. Regarding the situation of determining the surface area of each piece of cake, several responses are predicted to emerge. First, students immediately divide the total area of the cake (the area of the circle) by 8 to get the area of each piece. Second, students utilize the information they obtain about central angles. The teacher should ask students to explain the reasons for the answers. Another response that may arise in this situation is that students have difficulty finding comparative values or that students cannot even give an answer. Teachers might encourage students to discuss with friends. Teachers can also direct students to remember the concept of comparison and the elements of circles.

Lesson Design of Activity 2

Anticipated learning obstacles in the design of this activity include errors related to the concept of the relationship between the central angle, arc length, and area of the arc of a circle. The learning objectives contained in this design are for students to be able to determine the ratio between the sizes of the two central angles; the ratio of the lengths of the arcs; the ratio of the areas of the sectors; and conclude the relationship between the three.

An explanation of the concept of the relationship between the central angle, arc length, and circular cross-sectional area in this didactical design is presented through simple experimental activities as well as reinforcing the concept of value comparisons that have been obtained by students in previous learning experiences. Students are asked to do several similar experiments. This is so in order that students are able to observe the pattern that is formed. Furthermore, students are guided to be able to draw conclusions about the relationship between the central angle, the arc length, and the area of the circular section without having to stick to the formulas that have been given in the textbooks they have. There are two situations that will represent Lesson Design of Activity 2. Situation 1 in Lesson Design of Activity 2 will direct students to the concept of comparing two central angles of a circle, two arcs of a circle, and two edges of a circle through an activity of manipulating objects (Scheme 6).

Question 7

Do the following activities.

a. Draw a circle with center O with a diameter of 7 cm.
b. In this circle, make a central angle AOB 30º and a central angle COD 60º.
c. Measure arc AB and arc CD using a thread, then compare the results.
d. Trace the OAB mesh and cut around the OAB mesh. Then measure the OCD ring using the OAB wall. Pay attention to the comparison of the OAB wall with the OCD wall.
e. Check the comparison value between the two central angles, the length of the two arcs, and

Scheme 6. An example of questions for Situation 1 of lesson design of activity 2

Meanwhile, Situation 2 is given to direct students to be able to carry out the formulation and then verify it until they conclude that the ratio of the sizes of two central angles is equal to the ratio of two arc lengths and the ratio of two areas of the radius corresponding to these central angles (Scheme 7).

Question 8

Repeat the activity similar to Question No. 7, but with a different pair of angles. What can you conclude?

Scheme 7. An example of questions for Situation 2 of lesson design of activity 2

The responses that emerge are that students have difficulty in expressing ideas to arrive at the right conclusions. This might occur because students are not used to expressing ideas or concluding something. However, after being given some assistance or scaffolding, students can finally come to the right conclusion. This means that potential development has been achieved by students.

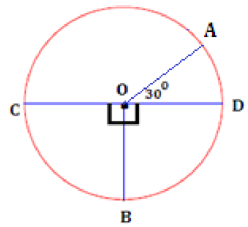
Lesson Design of Activity 3

Anticipated learning obstacles in the design of this activity include errors related to the connection of concepts in circle materials with other mathematical material concepts and students' difficulties in solving problem-solving questions. The learning objectives contained in this design are for students to be able to solve problems related to the application of the relationship between the central angle, arc length, and area of the arc of a circle.

In implementing the Lesson Design of Activity 3, students need quite a lot of help. Students seem not accustomed to dealing with questions that are not routine. In the implementation of situation four in the third activity of a hypothetical didactic design, students do not think of connecting the concept in circle material with the concept of the number of angles in a triangle, even though it is needed. After being given assistance, it will turn out that some students begin to understand and think about using this relationship in solving problems. This finding prompts the researchers to add instructions to help students better understand the situation in an empirical didactical design. It is also found that students have difficulty solving the given problem because they fail to describe the problem or construct a mathematical model of the given situation. However, students who can describe the problem correctly tend to be able to solve the problem. To help the students, commands to make a sketch of the situation or problem given in the empirical didactical design are given. The following problems or questions are developed to represent Lesson Design of Activity 3 for students' practice.

Question in Situation 1 is raised to encourage students to solve problems without relying on formulas. Students must really understand the relationship between central angles, arc lengths, and radius areas to be able to solve this problem well. Apart from that, students must also understand that the length of the arc is part of the circumference of the circle and the area of the circle is part of the area of the circle (Scheme 8).

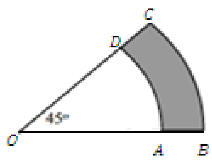
Question 9
Look at the picture below. It is known that the angle AOD is 30º and the arc length BC is 11 cm. Determine the area of the AOD mesh.



Scheme 8. An example of question for Situation 1 of lesson design of activity 3

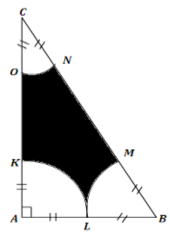
Following Situation 1, there is Situation 2 where students are given questions to help them practice in connecting mathematical concepts about circles and other mathematical concepts. Scheme 9 shows some examples of questions that represent this situation.

Question 10
The following figure is a circle with a central angle of 45º. If the length of OA= 42 cm and AB= 14 cm, determine the area of the shaded region.



Question 11
On Sunday morning, Rahmat ran circling Hotel Indonesia roundabout for 5 1/8 laps. It is known that the diameter of the Hotel Indonesia roundabout is 100 meters and Rahmat's distance from the edge of the roundabout while running is 2.5 meters. Determine the distance Rahmat has traveled.

Question 12
The following picture is a model of a garden in the form of a right triangle with a right angle at A. Corner points A, B, C are the center of a circle with the same radius of 3 m and length AC is 8 m. Determine the perimeter of the shaded area.



Scheme 9. An example of question for Situation 2 of Lesson Design of Activity 3

And the last situation, Situation 3, is given to familiarize students in working on problem-solving questions, especially in the form of story problems (Scheme 10).

Question 13
A 2 x 2 m is made in the middle of a square grass field with an area of 100 m^2. A goat is tied to it using a 2 m long rope to one corner outside the cage.
a. Describe the situation as well as the measurements.
b. Determine the maximum area that the goat can explore to eat grass if the goat cannot enter the cage.

Scheme 10. An example of question for Situation 3 of lesson design of activity 3

DISCUSSION

The development of a didactical design is carried out through three stages of the thinking process, the first phase of which is being the focus of the current study (Prediger et.al., 2015). The first phase is a stage of thinking before learning to obtain a hypothetical didactical design along with predictions of responses and anticipation. This designing process is conducted to strengthen students' understanding regarding circle elements, including the central angles of the circle and arcs and sectors of the circles.

The designed design contains several didactical situations along with predictions of student responses and their anticipations. The didactical situation presented emphasizes several aspects, namely aspects of action, formulation, validation, and institutionalization in classroom learning (Suherman, 2003; Alves et al., 2021). Based on these actions, it is hoped that a situation will be created that will become a source of information for students so that the learning process occurs. In this learning process, students take action on existing situations so as to create new situations which will then become information for the teacher. The teacher's follow-up action in response to students' actions towards the previous didactic situation will create a new didactical situation. From these situations, it is hoped that students will be able to make a formulation of the actions that have been taken, for example making patterns and generalizing them. Next is validation which aims to confirm the results of students' thinking. A series of situations experienced by students during the learning process must belong to the students so that students can solve problems in a different context than those that have been discussed in learning; this is what is called institutionalization (James et al., 2018).

Based on the results of the analysis of students' answers, the results of the analysis of teaching materials, and the results of interviews, as well as considering learning obstacles, learning trajectories, didactic situation theory and supporting learning theories, the researchers developed a hypothetical learning trajectory design for learning circle material in junior high school, especially on the topic of relationships among central angles, arc lengths, and sector areas of the circle. The design begins with strengthening students' understanding of related material that students have previously acquired, including the elements of a circle, the concept of the circumference and area of a circle, as well as the concept of equivalent proportion.

The elements of a circle that are closely related to this discussion are the central angle of a circle and the arc and sector of a circle. The design is formed in such a way as to strengthen students' understanding of these elements in a comprehensive manner. Students must understand that an arc of a circle is part of a circle and a sector of a circle is part of the area bounded by the circle so that the arc length is part of the circumference of the circle and the sector area is part of the area of the circle. The introduction of the concept of the relationship between central angles, arc lengths and the sector area of a circle in this didactic design begins with strengthening the concept of equivalent proportion that students have acquired in previous learning experiences. Next, students are guided to be able to draw conclusions about the relationship between central angles, arc lengths, and the area of a circle without having to stick to the formulas given in their textbooks. The design contains several didactic situations along with predictions of student responses and their anticipation. The didactic situation emphasizes several aspects, namely aspects of action, formulation, validation, and institutionalization in classroom learning. The learning process begins with an activity carrying out an action, namely presenting contextual problems. Based on these actions, it is hoped that a situation can be created that becomes a source of information for students so that the learning process occurs. In this learning process, students take action on existing situations so that new situations are created which will then become information for the teacher. The teacher's continued action in response to the student's action in the previous didactic situation will create a new didactic situation. From these situations, it is hoped that students will be able to make a formulation of the actions they have taken, for example creating patterns and generalizing them. Next is validation which aims to confirm the results of students' thinking. A series of situations experienced by students during the learning process must belong to the students so that students can solve problems in a different context from what has been discussed in learning, and this is what is called institutionalization (Sidik et al, 2023).

Some of the difficulties faced by students in implementing the three learning activities conducted in this study can be partly overcome by the interaction of students with teachers or with other students. This is related to the two implications of Vygotsky's theory in learning. First, it is desired that the classroom setting be in the form of cooperative learning between groups of students with different abilities, so that students can interact in doing difficult tasks and bring up effective problem-solving strategies in their respective nearest/proximal development areas. Second, Vygotsky's approach to learning emphasizes scaffolding. With scaffolding, the longer will students be able to take responsibility for their own learning (Suherman, 2003). Another theory that might support the didactical design is the fact that student's cognitive development will develop chronologically along with the age development (Sagala, 2013). Hence, teachers might want to encourage students to discuss with friends. Teachers can also assist in directing students to recall the concept of comparison and also the elements of circles.

Since this study only focuses on the first phase of this didactical design, which is the preparation phase; hence further studies following the results of this study should be conducted by revising the design, implementing it in another classroom, and doing retrospective analyses.

CONCLUSION

The hypothetical learning trajectory is developed based on the results of preliminary studies which include identification of learning obstacles as well as theoretical analysis and repersonalization. Based on this series of analyses, a hypothetical learning trajectory about arc lengths and sector areas can be developed. The HLT is designed using a realistic mathematics education (RME) approach. There are three anticipated learning obstacles in the design of the activities, including errors related to the concept of the relationship among arc lengths, circle circumferences, sector areas, and circle areas; errors related to the concepts of the relationship among the central angles, arc lengths, and sector areas of a circle; as well as errors related to the connection of concepts in circle materials with other mathematical material; and students' difficulties in problem-solving questions.

REFERENCES

Alves, F. R. V., Silva Camilo, A. M. D., Fontenele, F. C. F., & Catarino, P. M. M. C. (2021). Didactical engineering in the conception of a teaching situation originated from Brazil's SPAECE assessment with the support of the GEOGEBRA software. *Acta Didactica Napocensia*, *14*(2), 84-98. https://doi.org/10.24193/adn.14.2.7

Anggraini, R.S., Fauzan A. (2020). The effect of realistic mathematics education approach on mathematical problem-solving ability. *EDUMATIKA: Jurnal Riset Pendidikan Matematika*, 3(2), 94-102. https://doi.org/10.32939/ejrpm.v3i2.595

Baroody, A. J., Clements, D. H., & Sarama, J. (2022). Lessons learned from 10 experiments that tested the efficacy and assumptions of hypothetical learning trajectories. *Education Sciences*, *12*(3), 195. https://doi.org/10.3390/educsci12030195

Diana, Suryadi, & Dahlan. (2020). Students' creative thinking skills on the circle subject in terms of learning obstacle and learning trajectory. *Journal of Physics: Conference Series*, *1521*(2020), 032084. https://doi.org/10.1088/1742-6596/1521/3/032084

Duyen, N. T. H., & Loc, N. P. (2022). Developing primary students' understanding of mathematics through mathematization: A case of teaching the multiplication of two natural numbers. *European Journal of Educational Research*, *11*(1), 1-16. https://doi.org/10.12973/eu-jer.11.1.1

Hasibuan, A. M., Saragih, S., & Amry, Z. (2019). Development of learning materials based on realistic mathematics education to improve problem solving ability and student learning independence. *International electronic journal of mathematics education*, *14*(1), 243-252. https://doi.org/10.29333/iejme/4000

James, F., & Augustin, D. S. (2018). Improving teachers' pedagogical and instructional practice through action research: Potential and problems. *Educational Action Research*, *26*(2), 333-348. https://doi.org/10.1080/09650792.2017.1332655

Lady, A., Utomo, B. T., & Lovi, C. (2018). Improving mathematical ability and student learning outcomes through realistic mathematic education (RME) approach. *International Journal of Engineering and Technology*, *7*(2), 55–57. https://doi.org/10.14419/ijet.v7i2.10.10954

Mayer, R. E. (2001). What good is educational psychology? The case of cognition and instruction. *Educational Psychologist*, *36*(2), 83-88. https://doi.org/10.14419/ijet.v7i2.10.10954

National Council of Teachers of Mathematics. (2020). Principles and Standards for School Mathematics. Reston: National Council of Teachers of Mathematics. https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/

Plomp, T. (2013). *Educational Design Research: An Introduction. In: T. Plomp & N. Nieveen (Eds.), Educational Design Research Part A: An Introduction* (pp. 10-51). Enschede: SLO.

Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes: An overview on achievements and challenges. *ZDM Mathematics Education*, *47*(6), 877-891. http://dx.doi.org/10.1007/s11858-015-0722-3

Sagala, S. (2013). *Konsep dan Makna Pembelajaran [Concepts and Meaning of Learning].* Bandung: Alfabeta.

Serin, H. (2018). Perspectives on the teaching of geometry: Teaching and learning methods. *Journal of Education and Training*, *5*(1), 131–137. https://doi.org/10.5296/jet.v5i1.12115

Sidik, G. S., Zahrah, R. F., Hariyani, M., & Fadhilaturrahmi, F. (2023). Development of mathematics teaching materials using didactical design research: A study on enhancing pedagogical content knowledge in quadratic inequalities. *Journal of Education for Sustainable Innovation*, *1*(1), 39-48. https://doi.org/10.56916/jesi.v1i1.489

Simon, M.A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114–145. https://doi.org/10.2307/749205

Suherman, E. (2003). *Strategi Pembelajaran Matematika Kontemporer [Contemporary Mathematics Learning Strategies]*. Bandung: Jurusan Pendidikan Matematika FMIPA UPI.

Suwarsono. (2001). *Pendidikan Matematika di Indonesia [Mathematics Education in Indonesia].* Jakarta: Depdiknas.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). *Realistic mathematics education. In: Encyclopedia of mathematics education* (pp. 713-717). Cham: Springer. https://doi.org/10.1007/978-3-030-15789-0_170

Van Eerde, H. A. A. (2013). Design research: Looking into the heart of mathematics education. In: *Proceedings van de 1st South-East Asian Design Research conference*, 2-23 April 2013, UNSRI, Palembang (pp. 1-11). https://dspace.library.uu.nl/handle/1874/289586