

Number sense profile of prospective elementary school teachers in blended Mathematics learning

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ABSTRACT

Number sense is a skill that contributes significantly to learning mathematics. However, number sense is often positioned as a fundamental skill whose development is more focused on children. The contribution of number sense in mathematics is even more apparent at higher levels of education. Ironically, number sense seems ignored and has become a rarely studied topic in higher education. Thus, the student's number sense ability profile seems buried with various problems. This study aims to reveal the profile of prospective elementary school teachers' number sense abilities and the factors causing their failure in solving math problems during the implementation of blended learning. This study uses a qualitative approach with 37 prospective elementary school teachers as research subjects. The observation of the test and lecture activities is the data collection method. The results showed that the prospective elementary school teachers' number sense ability was still relatively low. Based on the components, the order of number sense abilities is from the highest, namely: (1) knowledge and number facilities with the achievement of 14.41%; (2) knowledge and facilities for number operations with the achievement of 8.12%; and (3) knowledge and facilities of numbers and operations for computing settings with an achievement of 1.8%. The low number sense ability is caused by the habit of solving problems procedurally and the failure to solve problems due to misconceptions, not understanding concepts, inaccuracy, inability to understand questions, and difficulty representing fractions. Various factors causing the failure arose due to the limitations of implementing blended learning.

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INTRODUCTION

Numbers are a material that will never be separated from the study of mathematics. As the main concept of mathematics (Purnomo et al., 2014), the number is a mandatory study material at every level of education, from basic to higher education. Nevertheless, the complexity of numbers is still one of the difficult mathematical studies to conquer (Güven & Çolak, 2019; Khairani & Shamsuddin, 2021; Namkung et al., 2018). One of the reasons is the lack of mastery of number sense (Lee et al., 2021; Yang & Sianturi, 2019, 2021). As a result, solving mathematical problems is often only interpreted as a form of procedural computing without being accompanied by the development of logic to explore the concept.

Previous studies have widely revealed the importance of number sense in mathematics learning (Er & Artut, 2018; Whitacre, 2018; Yang & Sianturi, 2019). Number sense is associated with the ability to understand the concept of numbers and their operations---including mental

computational---to solve mathematical problems flexibly, effectively, and efficiently (Hinton et al., 2015; Purnomo et al., 2014; Reys et al., 2009; Yang & Sianturi, 2021). Solving problems using the number sense concept is expected to prevent students from the trap of procedural completion that relies on formulas and systematic computational processes. This hope is in line with Yang and Wu (2010) that excessive pressure to perform written procedural computations hinders the development of number sense abilities, mathematical thinking, and understanding. Thus, it is unsurprising that number sense is often used to predict mathematical ability (Gerzel-Short & Hedin, 2022; Voronin et al., 2018). In other words, the number sense ability profile can also be used to identify mathematics difficulties.

Referring to its role and benefits, the development of number sense should be one of the priorities in learning mathematics. As a type of mathematical skill, developing number sense requires consistent and continuous relevant activities and assignments (Ben-Yehuda & Sharoni, 2021; Can & Yetkin Özdemir, 2020). This habit can be realized by providing HOTS-based practice questions that can strengthen understanding of concepts, hone problem-solving skills, and build mathematical logic, the basic skills to practice number sense. However, the facilities for training on such questions are still not evenly distributed in Indonesia. In this context, elementary to senior high school students tend to be more facilitated by the Minimum Competency Assessment (MCA), which has been promoted since 2021. On the other hand, students in college who carry out learning with a more flexible curriculum are less facilitated to get practice questions that lead to number sense ability development, whereas students studying at the education faculty are prospective teachers who are required to be able to teach the material well to their students.

The Elementary School Teacher Education study program is a part of the education faculty that seeks to prepare the best prospective teachers. Talking about the development of number sense abilities, the learning observations in the Elementary School Teacher Education study program, Universitas PGRI Yogyakarta (UPY), show that the mathematics learning that has been carried out has led to the development of number sense abilities. However, the teaching pattern is mostly focused on concept reinforcement which is the basic skill in developing number sense abilities. The mathematical concept was not maximally measured and facilitated during the pandemic due to the limitations of space, time, and interaction during the implementation of distance learning. Even though lecturers have facilitated the provision of varied learning support platforms---Google Classroom, Zoom, WhatsApp Group---with a combination of online, blended, and hybrid learning modes, communication and learning interactions have not been effective. Communication via WhatsApp Group and Google Classroom is dominated by student responses to learning information delivered, although occasionally, there are questions regarding the material. In addition, learning is held with several changes in learning modes, making students and lecturers fight harder to adapt. The researcher also found misconceptions about the types of numbers and their operations during the diagnostic test. This condition shows that students' number sense abilities as prospective teachers need to be studied further.

The aforementioned background motivates researchers to research prospective teachers' number sense abilities profile. Although number sense is not a new topic in mathematical research, it must be acknowledged that most research on number sense has only focused on elementary and preschool levels (Ghazali et al., 2021; Güner & Gökçe, 2021). Besides, although number sense is often associated with early mathematical skills, it does not mean that students in college always have well-established number sense abilities. The proof is that many research results reveal low problem-solving abilities and misconceptions in various materials about numbers (Dagdag et al., 2021; Pentang et al., 2021; Powell & Nelson, 2021). Therefore, research on mathematical skills among students mostly focused on problem-solving or numeracy skills should complement research related to number sense. Number sense is one of the mathematical skills that contribute greatly to the success of solving mathematical and numeracy problems. In addition, the study of number sense among prospective teachers remains an important research topic because the research's results not only describe the number sense ability but can also reveal pro-

spective teacher learning constraints to do learning reflection and determining more effective number teaching strategies.

METHOD

This study uses a qualitative approach with a case study method (Creswell, 2009). This research was conducted in March-May 2022 when researchers carried out teaching practice activities at Universitas PGRI Yogyakarta. The subjects of this study were 37 prospective elementary school teachers from the Elementary School Teacher Education study program at Universitas PGRI Yogyakarta who came from the researcher's practical class. Procedurally, the research consisted of three stages (see Figure 1), namely: (1) data collection, (2) data analysis, and (3) conclusion. The research participants were anonymized, and the data collected from them were used only for research purposes. Thus, the participants' involvement in this study does not impact their academic performance.

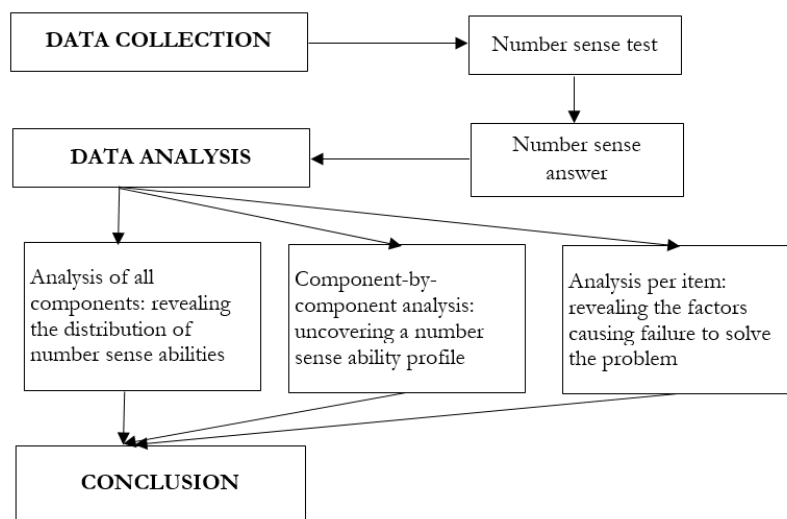


Figure 1. Research Procedure

The first step is data collection. There are two types of data collected through this research, namely: number sense ability data and learning activity data that have the potential to influence learning outcomes. Learning activity data was collected using the observation method. Number sense ability data was collected using a test method that focused on fraction material. The choice of fraction material is based on its characteristics that can accommodate the concept of the number system and its operations previously taught. Based on material experts' validation result, the number sense ability of this study was measured using three number sense components translated into eight indicators (Purnomo et al., 2014) as shown in Table 1.

Table 1. Components and Indicators of the Number Sense Test

Number Sense Component	Indicators	Item Test
Knowledge of and facility with numbers (A)	Sense of the order of place value on the number line	3
	Sense of the absolute and relative value of numbers	1
	Benchmarks system	7
Knowledge of and facility with operations (B)	Understanding the nature of operations in numbers	4
	Understanding the relationship between operation	5
Knowledge of and facility with numbers and operations to computational settings (C)	Understanding the relationship between the contexts of the problems and the appropriate computation	8
	Tendency to use a representation efficiently	2
	Tendency to review the data and the reasonable results	6

This number sense test consists of eight description questions and is carried out online through the Google form. The test procedure begins by emphasizing the importance of understanding the instructions for working on the questions, including (1) the work is carried out simultaneously for 20 minutes; (2) test work has emphasized the use of non-procedural methods (number sense approach), and (3) the collection of answers must include the working procedures carried out.

The second step is data analysis. Data analysis begins with correcting prospective teachers' test answers using scoring guidelines, as shown in [Table 2](#).

Table 2. Number Sense Test Scoring Guidelines

Indicators	Score
Students can solve problems correctly using the number sense concept	3
Students can solve problems correctly using procedural methods	2
Students can answer questions correctly, but the reasons are incomplete	1
Students' responses are completely wrong and cannot be interpreted	0

After the correction process, analysis is carried out based on the number sense component and the answer scores. Analysis per component number sense was conducted to determine the distribution and profile of prospective teachers' number sense skill and their difficulties in solving the test. The answer score analysis aims to reveal the factors causing prospective teachers' failure in solving problems. In this research, the factors causing failure of prospective teachers in solving problems were classified into five types, namely: misconceptions, lack of understanding of the concepts, inaccuracy, failure to understand questions, and difficulty representing fractions. The indicators of each type are shown in [Table 3](#). At the end of the process, all the results analysis be the basis for conclusions.

Table 3. Types of Causes of Prospective Teachers' Failure in Solving Number Sense Problem

Type of Causes	Indicators
Misconception	The answerer used solution concepts inappropriately
Lack of understanding of the concepts	The answerer was unable to apply the concept to solve the problem
Inaccuracy	The answerer made errors in reading questions and carrying out calculations
Failure to understand questions	The answerer's solution does not match what is asked for in the question
Difficulty representing fractions	The answerer was unable to make a correct fraction representation

FINDINGS AND DISCUSSION

Findings

Portrait of Blended Learning Implementation

The blended learning portrait aims to provide an overview of the implementation of learning before carrying out the number sense test. Data about the learning process was obtained using the observation method. Observation data collection is focused on obtaining information about how synchronous and asynchronous modes are carried out and how prospective teachers' activities carry out learning.

During the research period, mathematics learning was carried out blended with synchronous and asynchronous settings. In an asynchronous setting, the lecturer prepares material slides, explanation videos, and assignment sheets uploaded to Google Classroom for students. The material discussed in the lesson is the introduction of fractions consisting of definitions, meanings, and forms of fractions. To ensure that students learn, the lecturer gives assignments to make summaries and asks questions related to the material. As was customary in previous days, the lecturer gave a deadline of five days to complete this task. The summary evaluation results show a mean achievement of 50.14%, while the percentage of students who ask questions is 8.59%.

In a synchronous setting, learning is carried out using the Zoom Meeting platform. The learning material this time is related to fractional operations packaged in a classical discussion format, starting with the delivery of lighter material by the lecturer. In the learning process, the interactions that occur have decreased from the previous days. When the lecturer allows the students to ask questions, the students who ask are used to being active in previous lessons. Most other students tend to be silent and answer when the lecturer confirms their understanding of the material. At the end of the session, the lecturers and students agreed to carry out a number sense ability test on fractional material through the Google Form to end the synchronous arrangement through the Zoom meeting.

Profile Number Sense of Prospective Teacher

Table 4. Number Sense Ability's Distribution of Prospective Teachers

N = 37	Knowledge of and facility with numbers (A)	Knowledge of and facility with operations (B)	Knowledge of and facility with numbers and operations in computational settings (C)
Mean (%)	63.96	56.76	57.66
Standard deviation	1.11	0.99	0.86
Score 3 (%)	14.41	8.12	1.80
Score 2 (%)	6.31	12.16	18.92
Score 1 (%)	8.11	8.12	14.41
Score 0 (%)	71.17	71.62	64.87

Table 4 shows that the general achievement of knowledge of and facility with numbers (A) is the highest number sense component with a mean of 63.96%. The lowest achievement is in the knowledge of and facility with operations (B) component, with a mean of 56.76%. Judging from the variation of prospective teacher answers, component A has the most heterogeneous variation with a standard deviation of 1.11. Component C is the most homogeneous answer variation with a standard deviation of 0.86. Judging the level of prospective teacher's number sense ability in each component, Table 3 shows that the highest number sense ability (score 3) is in component A with a percentage of achievement of 14.41%; followed by component B with the achievement of 8.12%; and component C with an achievement of 1.80%.

Knowledge of and Facility with Number (A)

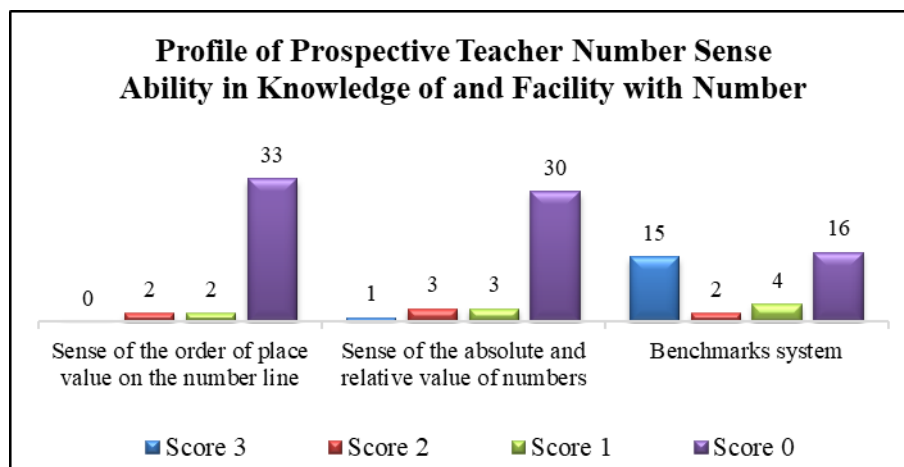


Figure 2. Profile of Prospective Teacher Number Sense Ability in Component A

Figure 2 shows that the prospective teachers' best ability is in the benchmark system as indicated by the 3 scores (solving with number sense) for as many as 15 prospective teachers (40.5%). From the aspect of sensitivity to absolute and relative values of a number, most pro-

spective teachers (81.1%) were completely unable to answer the questions correctly. The same thing applies to the aspect of sensitivity to the place value of numbers, with the percentage of wrong answers being 89.2%.

Prospective Teachers' Sensitivity to Place Values of Numbers

In this study, the sensitivity of the order of place value on the number line indicated knowledge and number facilities component with the lowest achievement. In this indicator, no prospective teacher could solve problems using the number sense concept. Only four prospective teachers (10.8%) believed there was a certain decimal between 0.23 and 0.24. There were two prospective teachers (5.4%) who answered that there were nine decimals, for example, 0.231; 0.232; ...; 0.239. Meanwhile, two other prospective teachers (5.4%) answered that there were 6 decimals without any examples. The rest, 89.2% of prospective teachers failed to solve the problem correctly. Judging from the variety of answers, the failure to solve this problem was caused by a misconception (see Figure 3) and the inability of prospective teachers to understand the problem (see Figure 4).

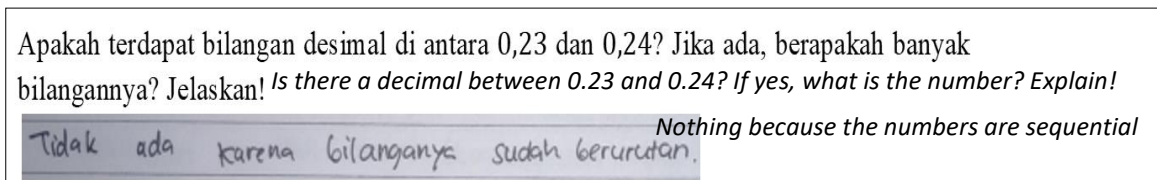


Figure 3. Indications of Misconceptions Answers to Question Number 7

The misconceptions that make incorrect answers in Figure 3 were caused by the generalization of using the concept of number order. The error was caused by adopting the integer-order concept to solve the decimals problem and their density properties. As a result, prospective teachers missed understanding the existence of an infinite decimal between two decimals, even though, at first glance, they look sequential.

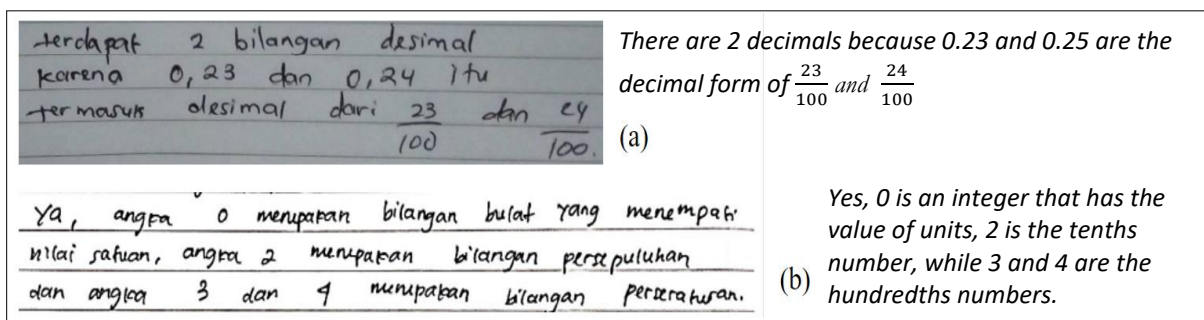


Figure 4. Answers Indicate Misunderstanding of Question Number 7

The incorrect answers in Figure 4 indicate a prospective teacher's lack of understanding of the question commands. If we look more closely, the answers in Figure 4(b) show a higher level of misunderstanding because when asked to show numbers between 0.23 and 0.24, the prospective teacher classifies the decimals to their place values. The answer in Figure 4(a) is considered to have a smaller level of understanding because prospective teachers missed the word "between", so they decided to rewrite the two shown decimal numbers.

Prospective Teacher Sensitivity to Absolute and Relative Values

Prospective teachers' sensitivity to absolute and relative values was measured by ordering several different types of fractions. In this question, only one prospective teacher (2.7%) could answer the question using the number sense concept (see Figure 5).

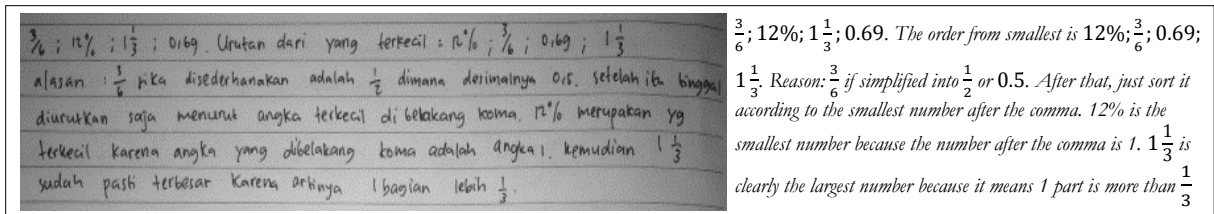


Figure 5. Example of Answers to Question Number 1 Using Number Sense Concept

There were six prospective teachers (16.2%) who could answer the questions correctly. However, three prospective teachers (8.1%) answered the questions without explanation, while the other three (8.1%) used a procedural method by changing all existing fractions into the same types of fractions and then comparing them. As a result, error answers experienced by 30 prospective teachers (81.1%) were dominated by incorrect answers that were incomplete (without any steps and reasons). A complete review of several answers indicates that misconceptions (see Figure 6) and inaccuracy (see Figure 7) were the dominant factors causing incorrect answers.

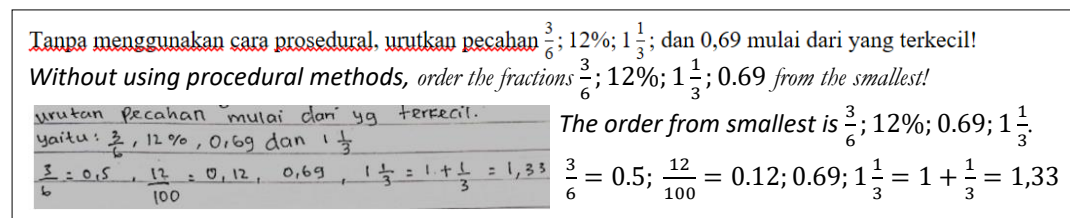


Figure 6. Indications of Misconceptions in Problem Number 1

Figure 6 shows that prospective teachers can do fractional conversions correctly. However, while ordering fractions, the prospective teacher's answer indicated a misconception in interpreting the value of decimals. The answer showed that the decimal value was measured based on the sum of the digits after the comma. $\frac{3}{6}$ was considered less than 12% because prospective teachers think that 0.12 was greater than 0.5. This misconception was similar to question 7, where prospective teachers used the large and small concepts in integers to compare decimals. Prospective teachers forgot that 0.5 could be changed to 0.50 when compared to 0.12.

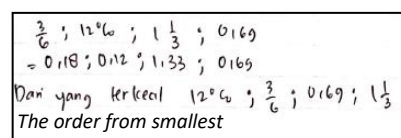


Figure 7. Indication of Inaccuracy in Problem Number 1

Figure 7 shows that prospective teachers can sort the fractions correctly after converting all fractions into decimals. However, the result of fractions conversion to decimal showed an error in converting $\frac{3}{6}$ to 0.18 (it should have been 0.5). This error was identified as a form of prospective teacher inaccuracy in computing because, in the same context, prospective teachers could change the $1\frac{1}{3}$ to 1.33 correctly.

Prospective Teacher Sensitivity to the Benchmarking System

Prospective teacher sensitivity to the benchmark system was measured through the ability to put a comma on the result of the multiplication of decimals. In addition, 15 prospective teachers (40.5%) who could solve problems using the number sense concept agreed to count the number of digits after the decimal point of the two numbers being multiplied (see Figure 8).

Tanpa menggunakan strategi operasi perkalian bersusun, tentukan posisi tanda koma pada hasil operasi $1234,5 \times 9,967 = 123042615$. Berikan alasanmu!

Without using the sequential multiplication operation strategy, determine the position of the comma in the result of the operation $1234,5 \times 9,967 =$

1234,5 x 9,967 = 123042615 karena dibelakang koma bilangan yang dikalikan terdapat 1 bilangan dan 3 bilangan. Maka hasil perkaliannya adalah dibelakang koma ada 4 bilangan.

1234.5 x 9.967 = 123042615, because behind the comma the number being multiplied there is 1 digit and 3 digits, so the result of the multiplication is 4 digits after the comma.

Figure 8. Example of Answers to Question Number 7 Using Number Sense Concept

Prospective teachers who got a score of 2 solved the problem procedurally with multiplication. Prospective teachers who had a score of 1 could answer questions correctly, but they did not give reasons. Prospective teachers who got a score of 0 were unable to answer the questions correctly because the solutions indicated a lack of questions understanding (see Figure 9) and a lack of understanding of concepts (see Figure 10). Another variation of answers was writing the product with an inappropriate comma without reasons.

Rasional, Karena tidak berulang-ulang dan berurutan. *Rational because it is not repetitive and sequential.*

Figure 9. Answers Indicate Misunderstanding of Question Number 7

Misunderstanding of the answers in Figure 9 is shown by the incompatibility between the answers and questions. The questions about placing a comma in decimals were answered by identifying the decimal type as a rational number. In addition, prospective teachers' understanding of rational numbers was incorrect because the reasons included (not repeated and sequential) were characteristics of irrational numbers.

3 angka dibelakang koma karena disamakan komanya

The three digits after the point are because the commas are equated.

Figure 10. Indication of Concept Misunderstanding in Problem Number 7

The answer in Figure 10 attempts to show that $1234,5 \times 9,967 = 123042,615$. The reason for the answer showed that prospective teachers could not understand the concept of decimal multiplication rules. The placement of the comma was adjusted to the sum of digits behind the largest comma owned by one of the decimals being multiplied. It should be adjusted to the sum of digits behind the comma in the two multiplied decimals.

Knowledge of and Facility with Operations (B)

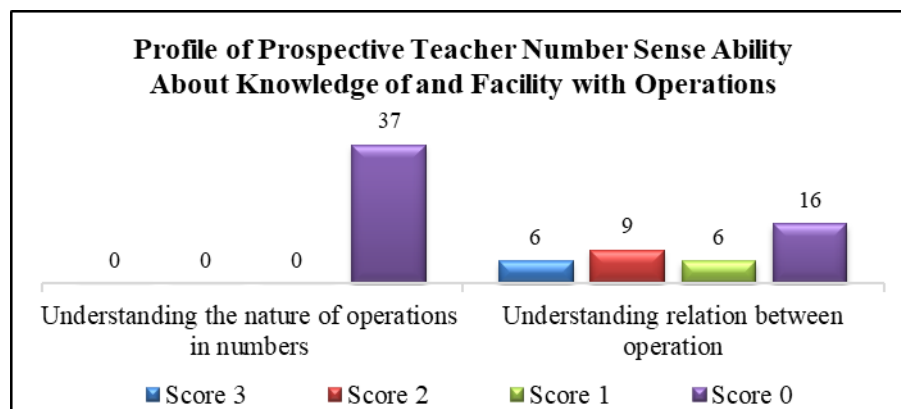



Figure 11. Profile of Prospective Teacher Number Sense Ability in Component B

Figure 11 shows that prospective teachers can better understand the relationship between number operations than the nature of number operations. The proof was that six prospective teachers (16.2%) successfully solved the problem using the number sense concept. On the other hand, none of the prospective teachers could solve problems on indicators of understanding the nature of number operations.

Prospective Teacher Sensitivity to Understanding the Properties of Number Operations

Prospective teachers' sensitivity to understanding the nature of number operations was tested to predict the results of the multiplication of A and B, each of which has no known exact value. This problem expected prospective teachers to understand the nature of the multiplication fraction, where the multiplication of fractions less than one always results in a smaller number. As a result, none of the prospective teachers could solve this problem. The results of the answer review showed that the misconception about the questions' command was the cause of the prospective teacher's failure to solve this problem (see Figure 12).

Perhatikan garis bilangan berikut ini! *Look at the following number line!*



Tentukan posisi $A \times B$ pada garis bilangan tersebut. Berikan alasanmu! *Determine the position of $A \times B$ on the number line! Give your reasons!*

A = 0,2 posisi A = 0,3 Menurut saya A terdapat pada posisi 0,2 sedangkan B pada 0,7 karena saya menghitung dengan pangkat aris.
 B = 0,7 posisi B = 0,6

(A) *I think A is in position 0.2 while B is at 0.7 because I counted with a ruler.*

(B) *Position A is close to 1 and far from 1 and less than $\frac{1}{2}$ too, so the possibility of its position is at the point $< \frac{1}{2} \rightarrow 0 < A < \frac{1}{2}$.*

(B) *Position B is closer to 1 and exceeds point $\frac{1}{2}$, it is possible that its position is at point $> \frac{1}{2} \rightarrow \frac{1}{2} < B < 1$.*

(C) $A = 0,1$ $A \times B = \frac{1}{10} \times \frac{3}{10} = \frac{3}{10}$
 $B = 0,3$

Figure 12. Answers Indicate Misconceptions of Command Question Number 4

Indications of misconceptions about questions' commands were based on the answer model of all test-taking prospective teachers. All answers were oriented to finding the values of A and B. Even the question needed to determine the products' position of A and B. Figure 12 shows the variation of answers in determining points A and B. Figure 12(A) shows the values of A and B obtained from the estimation results and measurement. Figure 12(B) essentially leads to a solution using a number sense concept without predicting the result. An increase in the completion pattern occurs in Figure 12(C). Although the solution process tended to be procedural, it was already in the form of multiplying A by B, but the answer could not be declared correct because the answerer did not mention the location of $A \times B$ on the number line as the question intended.

Prospective Teacher Sensitivity to Understanding the Relationship Between Number Operations

Prospective teachers' sensitivity to understanding the relationship between number operations was investigated by comparing the results of dividing fractions. In this indicator, prospective teachers' answers tended to be heterogeneous. The correct answer reached 56.7% with details, six prospective teachers (16.2%) answered using the number sense concept, nine prospective teachers (24.3%) answered correctly procedurally, and three others (16.2%) answered correctly without reason. Prospective teachers who answered questions using the number sense concept tended to use a comparison of the denominator of the divisor. They believed that a larger denominator of a divisor would result in a larger quotient (see Figure 13). The error of 16 prospective teachers (43.2%) in solving this problem was dominated by misconceptions (see Figure 14) and partly due to inaccuracy (see Figure 15).

Figure 14(A) shows that misconceptions are affected by the divisor of the two operations. When answering this question, prospective teachers were too focused on comparing the values of

the divisors without relating them to the possible outcomes. Figure 14(B) shows the misconception about procedural problem-solving. The value of 0.5 is obtained by operating $\frac{2}{3} \times \frac{3}{4}$.

Bandingkan hasil operasi bilangan pecahan antara $\frac{2}{3} \div \frac{3}{4}$ dan $\frac{2}{3} \div \frac{3}{7}$. Manakah yang lebih besar? Jelaskan jawaban Saudara tanpa mengubahnya ke dalam bentuk perkalian!

Compare the results of fractional operations between $\frac{2}{3} \div \frac{3}{4}$ and $\frac{2}{3} \div \frac{3}{7}$. Which one is bigger? Explain your answer without changing it into multiplication form!

$\frac{2}{3} : \frac{3}{4}$ dan $\frac{2}{3} : \frac{3}{7}$ yang lebih besar $\frac{2}{3} : \frac{3}{7}$, karena tercapat bilangan penyebut yang lebih besar yaitu 7, 7 lebih besar dan 4

Between $\frac{2}{3} \div \frac{3}{4}$ and $\frac{2}{3} \div \frac{3}{7}$ the bigger one is $\frac{2}{3} \div \frac{3}{7}$ because there is a bigger denominator, namely 7, 7 is bigger than 4

Figure 13. Example of Answers to Question Number 5 Using Number Sense Concept

(A) $\frac{2}{3} \div \frac{3}{4}$ dan $\frac{2}{3} \div \frac{3}{7}$
and
0,6 0,75 0,6 0,4

(B) $\frac{2}{3} \div \frac{3}{4}$ dan $\frac{2}{3} \div \frac{3}{7}$
Yang lebih besar yaitu $\frac{2}{3} \div \frac{3}{7}$ karena di ubah dalam bentuk pecahan hasilnya 0,5. Sedangkan $\frac{2}{3} \div \frac{3}{4}$ di ubah dalam bentuk pecahan hasilnya 0,2857.

Between $\frac{2}{3} \div \frac{3}{4}$ and $\frac{2}{3} \div \frac{3}{7}$ the bigger one is $\frac{2}{3} \div \frac{3}{7}$ because when converted to decimal the result is 0.5 while $\frac{2}{3} \div \frac{3}{4}$ when converted to decimal the result is 0.2857

Figure 14. Indications of Misconceptions in Problem Number 5

$$\frac{2}{3} + \frac{3}{7} = \frac{5}{10}, \text{ karena } \frac{5}{10} \text{ lebih besar dan } \frac{5}{7}$$

because $\frac{5}{10}$ is bigger than $\frac{5}{7}$

Figure 15. Indication of Inaccuracy in Question Number 5

Figure 15 shows the inaccuracy in reading the questions, where the division sign (\div) is seen as a plus sign (+). That finding showed that prospective teachers did not yet understand the concept of adding fractions, which requires the equations of the denominator.

Knowledge of and Facility with Numbers and Operations in Computational Settings (C)

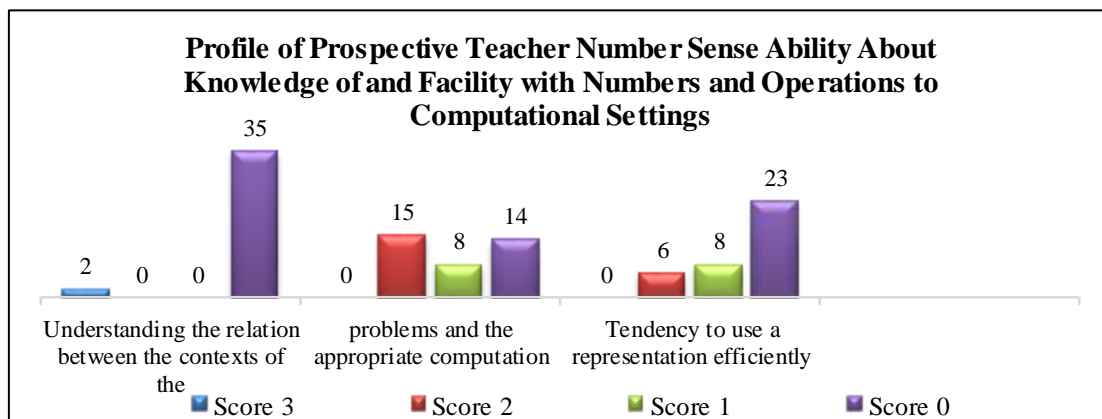


Figure 16. Profile of Prospective Teachers' Number Sense Ability in Component C

Figure 16 shows that the prospective teachers' best ability is in the indicator of understanding the relationship between the context of the problem and the correct calculation, indicated by the presence of two prospective teachers (5.4%) able to answer correctly using the number sense concept. In the indicator of the tendency to use representation efficiently, problem-solving was dominated by the procedural method, with an achievement of 40.5%. On the tendency of review data and rationale results, most prospective teachers (62.2%) were unable to solve the questions correctly, and no prospective teachers could answer questions using the number sense concept.

Prospective Teacher Sensitivity to the Relationship Between Problem Context and Calculations

Prospective teachers' sensitivity to the relationship between the context of the problem and their calculations was investigated using their skills in solving story problems. Only two prospective teachers (5.4%) were able to answer correctly using the number sense concept (see Figure 17). Another 35 prospective teachers (94.6%) could not solve the problem because they had difficulty representing $\frac{2}{3}$ (see Figure 18).


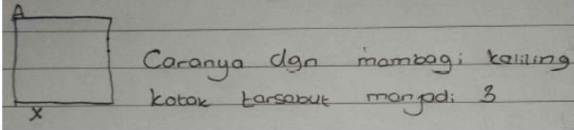
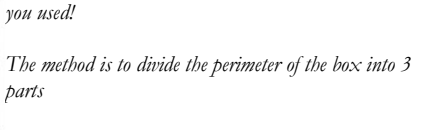
<p>Danira akan berjalan mengelilingi kolam renang berbentuk persegi sebanyak satu kali putaran. Danira mengawali langkahnya dari titik A, kemudian mengikuti anak panah. Berilah tanda silang pada persegi di bawah yang menunjukkan $\frac{2}{3}$ dari jarak total yang akan ditempuh Danira! Jelaskan strategi yang Saudara gunakan!</p> <p>A →</p> 	<p><i>Danira will walk around the square-shaped swimming pool for 1 lap. Danira started her steps from point A, then followed the arrows. Put a cross in the square below which shows $\frac{2}{3}$ of the total distance Danira will cover. Explain the strategy you used!</i></p> <p><i>The method is to divide the perimeter of the box into 3 parts</i></p>
	

Figure 17. Example of Answers to Question Number 8 Using Number Sense Concept

The idea of solving using the number sense concept was done by interpreting the square as a unit with a fractional value of $\frac{3}{3}$, so to determine its $\frac{2}{3}$ parts, prospective teachers divided the perimeter of the square into three parts, then placed the point in the second order.

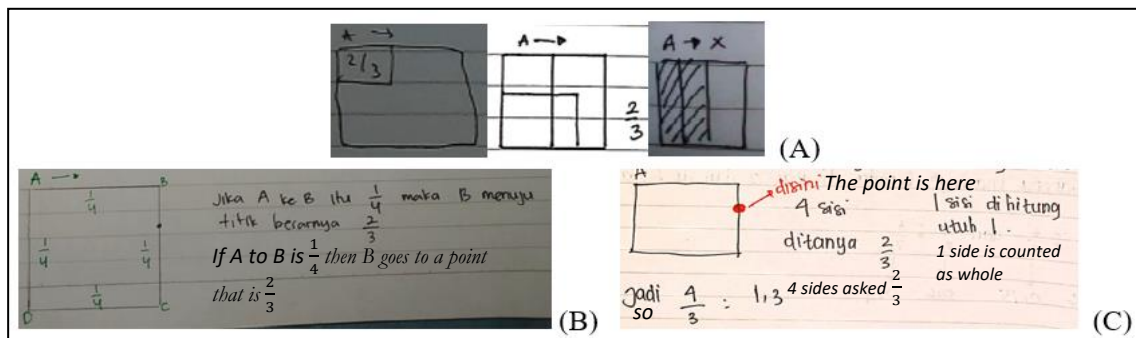


Figure 18. Answers Reflecting Difficulty Representing Fractions

Figure 18(A) shows that prospective teachers tend to represent $\frac{2}{3}$ in the context of surface area. The dominance of procedural thinking caused this condition because so far, the discussion of fractions has often focused on the division of fields. Figure 18(B) and Figure 18(C) essentially show prospective teachers' understanding that they are doing line division, so it appears that a certain point is believed to represent $\frac{2}{3}$. The error in Figure 18(B) occurs because prospective teachers fail to build logic that connects $\frac{1}{4}$ with $\frac{2}{3}$. As a result, the placement of the points was not appropriate. The error in Figure 18(C) occurs because the logic of the prospective teachers' thinking is interrupted in the middle of the road, so they put a point according to the quotient of 4 by 3. Even though to reach the point referred to in the problem, the quotient should be multiplied by the number 2, according to the numerator $\frac{2}{3}$.

Prospective Teacher Sensitivity to the Tendency to Use Representation Efficiently

The tendency to use representation efficiently was investigated through the prospective teachers' ability to find the relationship between two fractions by representing them. The results showed that 15 prospective teachers (40.5%) answered the questions correctly using the

procedural method, but no prospective teachers (0%) solved the questions using the number sense concept. Most of the 14 prospective teachers' failure (37.8%) in solving this problem was influenced by misconceptions (see Figure 19).

Di antara bilangan pecahan $\frac{5}{6}$ dan $\frac{9}{10}$, manakah yang paling mendekati 1? Mengapa? *Between the fractions $\frac{5}{6}$ and $\frac{9}{10}$, which is closest to 1? Why?*

Yang mendekati 1 adalah $\frac{9}{10}$ dan $\frac{5}{6}$ bisa dibolong sama. *The fraction closest to 1 is $\frac{9}{10}$ and $\frac{5}{6}$ can be said to be the same*

Jika $\frac{9}{10} + \frac{1}{10} = \frac{10}{10} = 1$ dan $\frac{5}{6} + \frac{1}{6} = \frac{6}{6} = 1$ *so*

(A)

~~$\frac{5}{6}$ lebih mendekati 1 karena penyebutnya lebih. Menurut saya sama saja karena baik $\frac{5}{6}$ maupun $\frac{9}{10}$ hanya berelish satu antara pembilang dan penyebutnya. Untuk mendapatkan hasil sendiri adalah dari pecahan yang pembilang dan penyebutnya memiliki angka yang sama.~~

I think it's the same because the numerator and denominator of $\frac{5}{6}$ and $\frac{9}{10}$ only differ by 1. To get a result of 1 the numerator and denominator must have the same number.

(B)

Figure 19. Indications of Misconceptions in Solving Problem Number 2

Figure 19(A) and Figure 19(B) show that prospective teachers can not choose a fraction closer to 1, because the analysis only focuses on the quantifier of the additional fraction that makes it worth 1. In this context, prospective teachers' sensitivity to effectively representing fractions started to be awakened. However, their negligence in seeing the additional fractions as a whole ($\frac{1}{10}$ and $\frac{1}{6}$) was proof of a misconception. Once again, the prospective teachers used the concept of integer comparison to compare fractions. As a result, prospective teachers assume that $\frac{1}{10}$ has the same value as $\frac{1}{6}$.

Prospective Teacher Sensitivity to Data Review Tendencies and Reasonable Results

The tendency to review data and reasonable results was identified through prospective teachers' ability to predict the results of number operations compared to certain numbers. The results showed that there were no prospective teachers (0%) who were able to solve problems using the number sense concept. Six prospective teachers (16.2%) answered procedurally, and 23 prospective teachers (62.2%) were unable to solve the questions correctly. An indication of a misconception caused the failure to solve the problem (see Figure 20).

Tanpa menyamakan penyebut, selidiki apakah operasi penjumlahan $\frac{4}{9} + \frac{3}{7}$ memiliki hasil kurang dari atau lebih dari 1? Jelaskan dengan contoh!

$\frac{4}{9} + \frac{3}{7} = \frac{7}{16}$ $7 < 1$

(A)

Habinya kurang dari 1 karena kedua pecahan nilainya sama sama kurang dari 1 sehingga ketika digumlahkan hasilnya akan kurang dari 1 pula.

Without equating the denominators, investigate whether the addition of $\frac{4}{9} + \frac{3}{7}$ has a result less than or more than 1! Explain with examples!

The result is less than 1 because both fractions have equal values less than 1, so when both of them are added, the (B) result is also less than 1.

Figure 20. Indications of Misconceptions in Solving Problem Number 6

Figure 20(A) indicates prospective teachers' misconceptions about performing fraction addition operations. In this case, the pattern of solving fractional operations was carried out using the concept of integer operations so that prospective teachers immediately added up numbers regardless of the difference in the denominators. Figure 20(B) indicates a misconception in understanding the nature of operations on fractions. In this context, prospective teachers used the properties of the multiplication of fractions to predict the result of fractional addition operations.

Discussion

Overall, the results of this study illustrate that the prospective teachers' number sense ability is still relatively low (Courtney-Clarke & Wessels, 2014; Senol et al., 2015). In terms of general achievements, the order of prospective teachers' number sense abilities was from the highest, namely: (1) Knowledge of and facility with the achievement of 63.96%; (2) Knowledge of and facility with numbers and operations to computational settings with the achievement of 57.66%; and (3) Knowledge of and facilities with the achievement of 56.76%. However, the actual order of number sense ability (score 3) consists of (1) Knowledge of and facility with an achievement of 14.41%; (2) Knowledge of and facilities with an achievement of 8.12%; and (3) Knowledge of and facility with numbers and operations to computational settings with an achievement of 1.8%.

In the knowledge of and facility with the number (A) component, the best number sense ability lies in the benchmark system indicator with an achievement percentage of 40.5%. The best number sense ability in the knowledge of and facility component lies in understanding the relationship between number operations, with an achievement percentage of 16.2%. In the knowledge of and facility component and its operation, the best number sense ability lies in the indicator of understanding the relationship between the context of the problem and the correct calculation, with an achievement percentage of 5.4%.

This study's findings indicate that prospective teachers' low number sense ability is caused by the habit of solving problems procedurally (Almeida et al., 2016; Yaman, 2015) and the inability of prospective teachers to solve problems correctly. The tendency to solve procedural questions is believed to be one of the inhibiting factors for developing number sense abilities (Yang & Wu, 2010). Procedural steps with systematic thinking processes are considered contrary to the characteristics of number sense which are identical to flexible thinking processes that rely on mathematical logic (Hinton et al., 2015; Reys et al., 2009; Yang & Sianturi, 2021).

Concerning the emphasis on flexible thinking processes in the number sense approach (Hinton et al., 2015; Purnomo et al., 2014; Reys et al., 2009; Yang & Sianturi, 2021), the researcher admits that the rules for problem-solving use steps and reasoning. Very different from most studies, several previous studies have attempted to reveal the ability of number sense using multiple choice or short answer models that do not require processing steps and reasons (Purnomo et al., 2014). Behind these differences, the design of the rules for conducting this research is considered more effective because it can train mathematical communication skills that are very much needed by prospective teachers, in addition to the main benefit of capturing the originality of the prospective teacher's flow of thought through the collected answer sheets. Based on the results of the study of the working process and prospective teacher reasons, the researchers found that the failure of problem-solving was caused by several factors, including misconceptions, lack of understanding of concepts, inaccuracy, failure to understand questions, and difficulty representing fractions.

The first is misconceptions. In this study, many misconceptions were found about using the concept of integers to solve fraction problems (Malone & Fuchs, 2017; Powell & Nelson, 2021). In the knowledge of and facility with the number (A) component, misconceptions occur regarding number order and decimal values (Lai & Wong, 2017; Roell et al., 2017). In the knowledge component and number operation facilities (B), misconceptions occur in terms of the concept of fractional division operations. The misconception in this context is the assumption that the bigger denominator of the divisor results in the bigger quotient. In addition, some prospective teachers work on the division of fractions using the rules of multiplication. In the knowledge of and facility component and its operation for computing settings (C), misconceptions occur in the incomplete meaning of fractions. Prospective teachers only pay attention to the numerator when comparing fractions (Aliustaoglu et al., 2018; Malone & Fuchs, 2017). In addition, misconceptions occur in adding fractions using the concept of adding integers (Powell & Nelson, 2021).

The second is not understanding the concept. The concept of misunderstanding in this study was found in the decimal multiplication rule (Powell & Nelson, 2021). Prospective teachers determine the location of the comma according to the sum of digits behind the largest comma owned by one of the decimals being multiplied.

The third is inaccuracy. In this study, there are two types of inaccuracy, namely inaccuracy in computing and inaccuracy in reading questions (Powell & Nelson, 2021). The computational inaccuracy is indicated by the inaccuracy of the conversion result of fractions into decimals, even though the results of the conversion of other fractions are correct. The inaccuracy in reading the questions occurs when prospective teachers see the divide sign (\div) as an additional sign (+).

The fourth is the failure to understand the problem (Dennis et al., 2016; Mädamürk et al., 2018). The lack of understanding of the questions was found in the knowledge of and facility component as well as the knowledge component and number operation facility. Prospective teachers' misunderstanding of the problem is marked by a form of completion that does not follow the question's expectations.

The fifth is the difficulty of representing fractions. The only difficulty in representing fractions is found in the knowledge of and facility component and its operations for computational settings. Indication of this difficulty is most visible when prospective teachers are supposed to represent fractions in the context of a line, but their representation is directed at the concept of a field.

The finding of the factors causing the failure to solve this problem certainly has a relationship with implementing blended learning during the research period. The lack of interactive discussions between lecturers and prospective teachers during the learning process (Zhou & Chua, 2016) is believed to be a factor in the incompleteness of concepts obtained by prospective teachers. As a result, the chances of prospective teachers experiencing misconceptions are getting bigger. The inaccuracy factor is caused by a short time limit for completing the test questions. This time is in stark contrast to the regular submission deadline of five days. Meanwhile, the misunderstanding about the questions was due to the absence of direct interaction between lecturers and prospective teachers during the test, so prospective teachers could not confirm the meaning of questions that were difficult to understand.

CONCLUSION

The distribution of pure number sense ability (score 3) from the highest, respectively, consists of (1) Knowledge of and facility with the number (A); (2) Knowledge and facilities for number operations; and (3) Knowledge of and facilities and their operations for computing settings. Sensitivity to the benchmark system is the best number sense ability in the knowledge of and facility component. In the knowledge component and number operation facilities, the best number sense ability lies in understanding the relationship between number operations. Meanwhile, understanding the relationship between the context of the problem and the right calculation is the best number sense indicator in the knowledge of and facility component and its operation. Quantitatively, the study results show that prospective teachers' number sense ability is still relatively low. The low number sense ability is influenced by the tendency of prospective teachers to use procedural steps to solve problems. In addition, the low number sense ability is also caused by misconceptions, not understanding concepts, inaccuracies, failure to understand questions, and difficulties in representing fractions, resulting in college prospective teachers' inability to solve problems correctly. Various factors causing the failure arose due to the limitations of implementing blended learning.

DISCLOSURE STATEMENT

The authors declare that they have no conflicts of interest to disclose concerning this article's research, authorship, and/or publication.

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